STUDY OF ESTUARINE POLLUTANT AND WATER QUALITY DISTRIBUTION IN THE NEW YORK CITY – NEW JERSEY METROPOLITAN AREA

SEPTEMBER, 1967

Prepared for

INTERSTATE SANITATION COMMISSION

NEW YORK • NEW JERSEY • CONNECTICUT

By
Quirk, Lawler and Matusky, Engineers
New York City

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WATER RESOURCES PLANNING
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SOLID WASTE DISPOSAL
AIR POLLUTION ANALYSIS

COMPUTER FACILITIES

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File: 144

September 26, 1967

Mr. Thomas R. Glenn, Jr.
Director & Chief Engineer
Interstate Sanitation Commission
10 Columbus Circle
New York, New York 10019

Dear Mr. Glenn:

A report of our study of the dispersal of pollutants in the New York-New Jersey estuarine complex is enclosed.

Models of two systems have been prepared: The Arthur Kill, including the bounding waters of Newark Bay, Raritan Bay and the Raritan River, and the lower Hudson River, including the bounding waters of the East River and Upper New York Bay.

Preliminary analysis showed that the transport of wastes between these two systems was small by comparison to the discharge, transport and decay of wastes within each system. For this reason, each system was evaluated independently of each other.

Development of the mathematical models and evaluation of the system parameters have been presented with much detail. This should facilitate use, by ISC personnel, of these techniques in evaluating other areas within the Compact boundaries.

Very truly yours,

John P. Lawler

John P. Lawler

JPL:bm

Preface

This report presents results of a study of the average behavior of several waterways in the network of estuaries around New York City and Northeastern New Jersey. Particular attention is paid the Arthur Kill and the lower Hudson River, and the Bays which bound them.

An attempt is made to show the improvement in water quality that could occur with upgraded levels of waste treatment. This report should be viewed as an interim statement. Additional investigation is needed. More data, particularly on waste loads, are required. The nitrification mechanism and the effect of thermal discharges should be included in the model. The model should be refined (time scale reduced) so that treatment requirements to satisfy "at any time" standards may be predicted directly from the model. Detailed recommendations for areas of further work are given in the report summary.

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Summary of Findings, Conclusions and Recommendations

1. One dimensional, steady state models of the Arthur Kill-Raritan River-Raritan Bay-Newark Bay complex, and of the lower Hudson River-East River-Upper New York Bay complex have been developed. These models yield tidal cycle, cross-sectional area-averaged concentrations of BOD and DO under equilibrium or steady state conditions.

2. The concentrations of BOD and DO generated by the model represent long term average conditions. The model gives moderately good agreement with the three month summer averages of 1964. During this period, river flows were relatively constant and the steady state condition was closely approximated.

3. Parameters which required numerical evaluation for use in the model included flow, dispersion coefficient, unit rates of BOD decay, atmospheric reaeration and tidal exchange, waterway geometry and waste loading. Evaluation of all parameters was made with the best available data.

4. Sensitivity analysis of the model showed that response of model generated DO was more sensitive to changes in total loading than to any other single parameter. For the Hudson, the DO varied linearly with load, and less strongly with reaeration coefficient, decay coefficient, flow and dispersion coefficient. Lack of sensitivity of the subject waterways to the system parameters is due to the distributed loading pattern, the high level of mixing, and to the influence of system boundaries.

5. Additional investigation of the loading pattern is necessary. Values of loads used in this study include estimates of untreated sources and grab sample measurements of some of the treatment plant effluents. Enumeration of sources, particularly on the New Jersey side of the Hudson River, may not be complete.

6. Individual DO measurements were as much as 25% saturation units below the long term averages in the Hudson River and as much as 20% saturation units below the long term averages in the Arthur Kill. Since these represent conditions which may occur "at any time," evaluation of treatment needed to meet stream standards included this effect. Further investigation of the variation of DO about the average is needed; since the number of measurements was small, the variation may be greater than actually observed. Secondly, treatment may be expected

to damp that fraction of the fluctuations due to variation in untreated discharges.

7. Total present loading on the Arthur Kill-Raritan Bay system is estimated to be 500,000 #BOD/day. Approximately 200,000 #/day of this can be assimilated without dropping the long term average DO below 4 ppm. The current policy of the Interstate Sanitation Commission requires 80% removal of all waste loads. At this level, the current standard of 30% DO saturation "at any time" should be met. On the basis of 80% treatment of present and future loads, and a long term minimum DO of 4.0 ppm, an additional future loading, 25% greater than the present load, may be carried.

8. Total present loading on the Hudson River-Upper Bay-East River complex is approximately 1,800,000 #BOD/day. 1,100,000 #/day of this load is discharged to the Hudson River and Upper New York Bay, between the New York-New Jersey state line and the Narrows.

   On the basis of present New Jersey DO standards, about 450,000 #/day can be assimilated by the Hudson River-Upper Bay system. Eighty per cent treatment of current sources from both states will reserve about 20% of this capacity for future use.

   On the basis of present New York DO standards, about 500,000 #/day can be assimilated by this system. Eighty per cent treatment of present sources will reserve about 30% capacity for future use.

9. The effect of nitrification on Hudson River DO was not considered. A preliminary estimate of this effect in the Arthur Kill shows that it may consume 20 to 40% of the available assimilation capacity in this waterway. Additional investigation of this effect should be made. Studies should include laboratory evaluation of nitrification on these estuarine waters before and after secondary treatment and inclusion of nitrification kinetics in the model.

10. A preliminary estimate of the effect of thermal discharges on Arthur Kill temperature and assimilation capacity was made. The discharge of an estimated 200 billion BTU daily may elevate the Arthur Kill temperature by some 2 to 4^{O}F. An increase in the ambient temperature from 76 to 81^{O}F entails a reduction in assimilation capacity of about 100,000 #BOD/day. Although cost

of cooling towers to offset this effect appears to be on the order of treatment costs, space requirements of cooling facilities may preclude their use. Further study of the thermal question should be made.

11. Raritan River model generated BOD and DO distributions did not yield good agreement with measured profiles. Raritan River cross-sectional area was assumed constant for model purposes but varies markedly from a maximum at its mouth in Raritan Bay to a minimum at the Fieldville Dam. Consideration of this variation in a refined model of the Raritan River would yield better agreement with measured data. Development of a refined Raritan River model would permit assessment of the effect of Raritan Bay effluents and upper Raritan River discharges on the DO profile in this River.

I. Report Objectives, Scope and Format

This report presents, to the Interstate Sanitation Commission, an analysis of pollution in the New York-New Jersey estuarine complex. Report objectives include the development of a mathematical model to describe the transport of BOD and dissolved oxygen in this area, and presentation of this development in sufficient detail to enable ISC personnel to use the analytical techniques in other studies of pollutant and water quality behavior within the Compact boundaries.

Study scope was limited to description of pollutant and water quality concentrations in terms of averages across a cross-section and over a tidal cycle. For the most part, steady state solutions only are developed. Results, therefore, should be interpreted in terms of monthly average behavior since the rapidity at which the system responds to external changes is not known.

Waterways investigated include the Arthur Kill, the Raritan River and Raritan Bay. Results obtained in previous studies (1), (2) on the lower Hudson River are also reviewed and expanded.

The report is formatted as follows. A brief outline of study results and the analytical procedures adopted to develop these results is given first. Following this, a detailed, step-by-step development is given. This development begins with the construction of the differential equations which describe estuarine pollutant transport. Selection of the numerical values of the parameters, which control the transport, such as flow, decay rate and exchange coefficient, are given next.

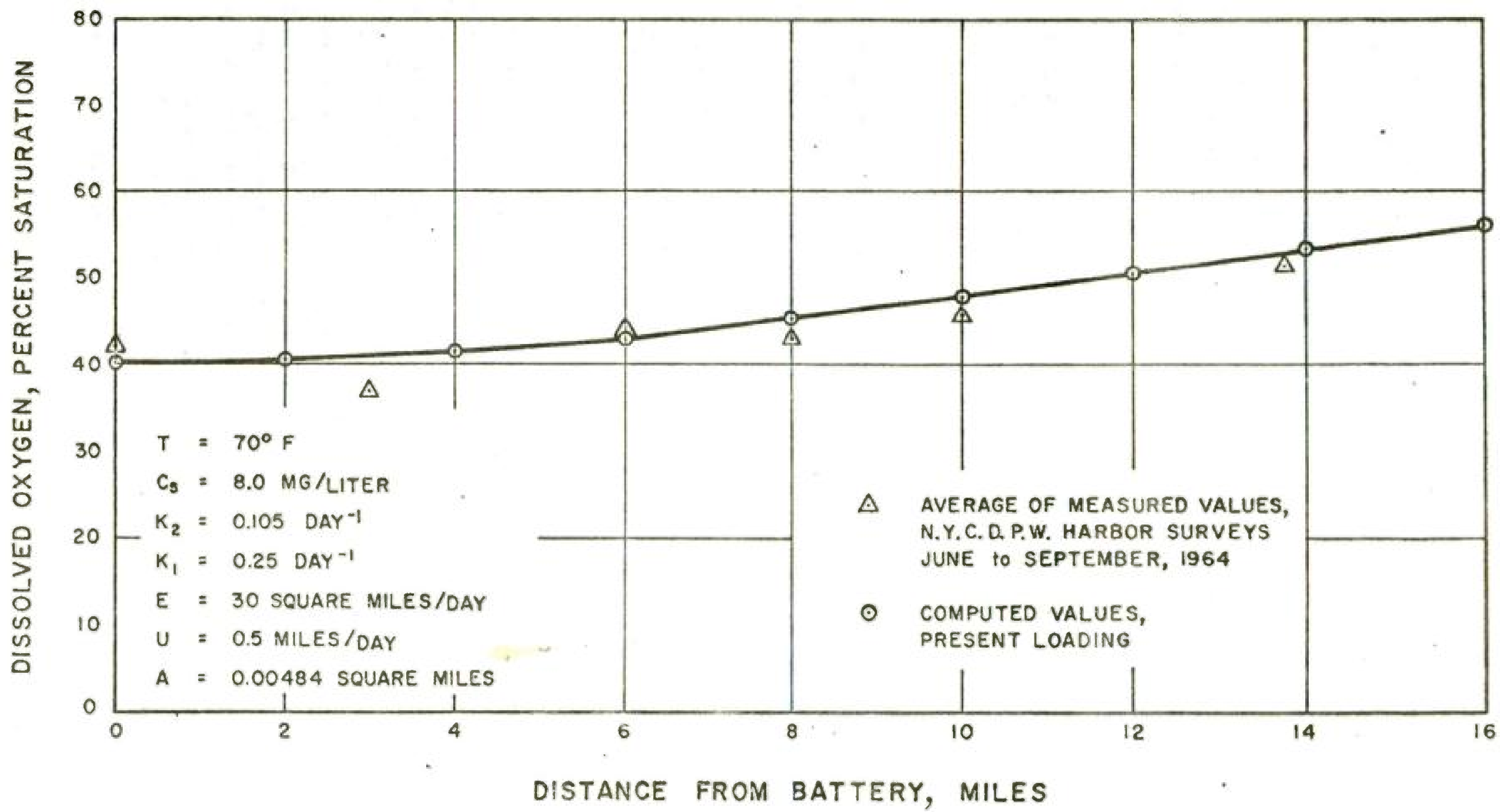
Following this, a simple model for the distribution of BOD and DO caused by discharge of a single waste source in an estuary of infinite extent is given, and numerical evaluation of the effect of some typical waste loads in the subject waterways is made. This simple model is an important step in the development of the multi-source, multi-waterway model utilized to describe pollution in the subject area. The basic boundary conditions and general solution of the defining differential equation, required items regardless of model complexity, are introduced at this juncture. Solution for the case of the single waste source establishes a quantitative cause and effect relationship between waste load, and river BOD and DO, and demonstrates the influence of flow, dispersion, decay and reaeration on this response of BOD and DO to load.

Development of a generalized model for a multi-source, multi-waterway system is given last. Solutions describing the distribution of BOD and DO, for conditions of present and future loading, are developed for the Arthur Kill-Raritan River and the Hudson River-East River complexes.

Preliminary results for distribution of temperature resulting from the discharge of thermal effluents to the Arthur Kill are given.

HUDSON RIVER ASSIMILATION CAPACITY ANALYSIS

COMPARISON OF MEASURED AND COMPUTED DISSOLVED OXYGEN DISTRIBUTIONS



II. Preview

Figure 1 shows a typical result obtained by means of the analytical approaches employed in this study. Dissolved oxygen concentrations in the lower Hudson River, between the Battery and the New York City-Yonkers line, are shown for present conditions.

The computed DO concentrations are obtained from a mathematical model of the estuary and represent area-averaged, tidal-smoothed values reached under equilibrium conditions. The measured values approximate these conditions and show good agreement with the computed values.

This agreement establishes the ability of the model to represent estuarine water quality characteristics. The model may then be used to predict river quality under other sets of conditions; for example, for general upgrading of waste treatment throughout the area.

Figure 2 describes the major steps involved in constructing the analytical representation of estuarine mass transport. The development begins by writing a mass balance on the rate of movement of pollutant through any small segment of the estuary.

The volume of this segment is permitted to approach an infinitesimal size. This procedure yields a differential equation which describes the net rate of pollutant transport at any point within the system.

This equation is then integrated to obtain the "general solution." Here it is recognized that the differential equation of estuarine mass transport describes the behavior for many different estuarine systems. Its general solution contains the particular solution for any of these; i.e., the general solution contains two as yet undetermined integration constants. These constants, when evaluated, will yield the particular solution for a given system.

For a particular system, these constants are determined by fitting the general solution to the boundary conditions which describe that system. The number of boundary conditions required is equal to the number of integration constants; the "fit" is obtained by substituting the general solution into the boundary conditions and solving a set of simultaneous equations for the integration constants.

The resulting particular solution will describe the distribution of pollutants, such as BOD, or water quality characteristics, such as DO, in the particular waterway being investigated. Since these profiles depend on waste loading and river characteristics, the behavior for various loadings and river conditions may be obtained.

Figure 3 gives a brief schematic of the above approach and shows the points at which physical or mathematical descriptions are employed. Figure 4 portrays the approach applied to transport of BOD in a fresh water stream. The equations shown are less complex than those for estuarine transport since longitudinal mixing, normally a negligible contribution in stream transport, has not been included.

MATHEMATICAL ANALYSIS OF RIVER AND ESTUARY POLLUTION

OBJECT: DETERMINE POLLUTANT CONCENTRATION PROFILES

DIFFERENTIAL EQUATION APPROACH

MATERIAL BALANCE

1. CONSIDERS RATE OF POLLUTANT TRANSPORT
2. WRITTEN OVER ANY FINITE SEGMENT OF ESTUARY

LIMIT AS SEGMENT SIZE SHRINKS

DIFFERENTIAL EQUATION

1. DESCRIBES POINT RATE OF POLLUTANT TRANSPORT
2. VALID AT ANY POINT WITHIN SYSTEM

INTEGRATE

BOUNDARY CONDITIONS

1. DESCRIBE BEHAVIOR AT RIVER BOUNDARIES
2. INTRODUCE EXTERNAL LOADS TO SYSTEM

GENERAL SOLUTION

1. YIELDS GENERAL SHAPE OF POLLUTANT PROFILE
2. IDENTICAL FOR ALL WATERWAYS DESCRIBED BY SAME DIFFERENTIAL EQUATION
3. POSSESSES AS YET UNDETERMINED INTEGRATION CONSTANTS

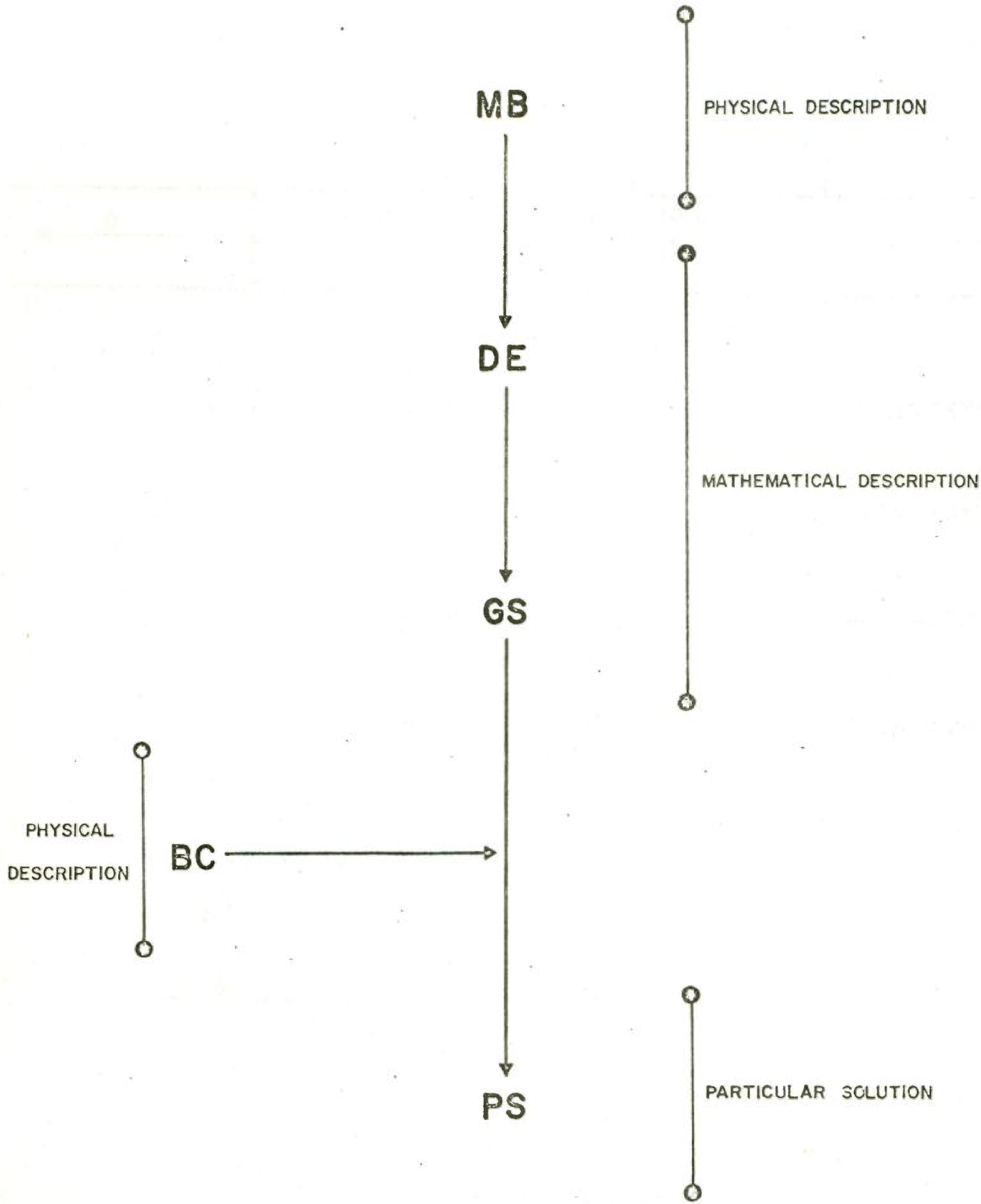
FIT GS TO BC

PARTICULAR SOLUTION

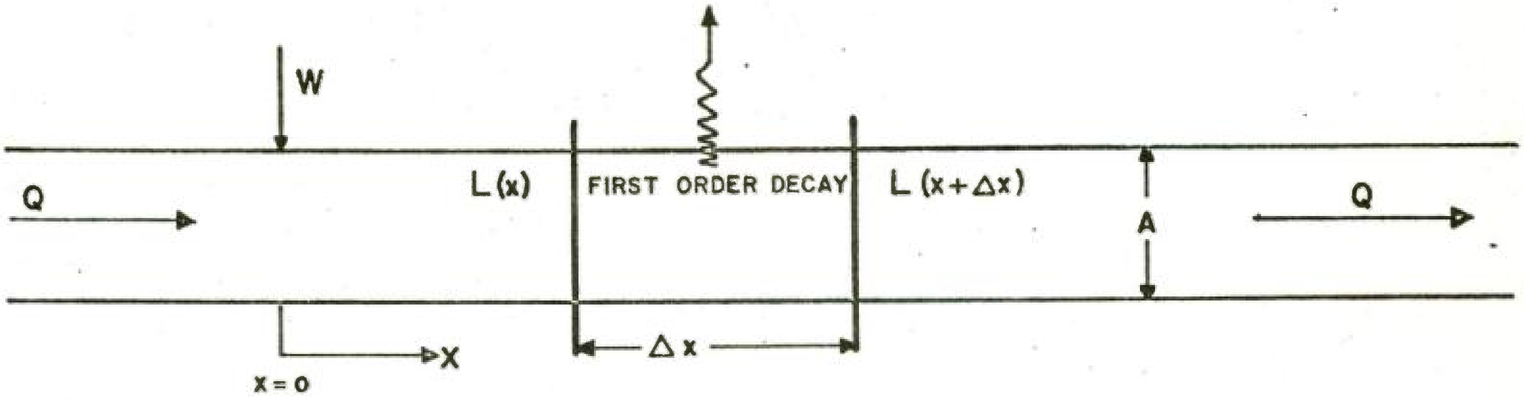
1. YIELDS PRECISE VALUES OF POLLUTANT CONCENTRATION
2. SPECIFIC TO SYSTEM UNDER INVESTIGATION
3. MAY BE EMPLOYED TO EVALUATE SYSTEM BEHAVIOR

FIGURE 3

DIFFERENTIAL EQUATION APPROACH



EXAMPLE: DECAY OF BOD IN A STREAM



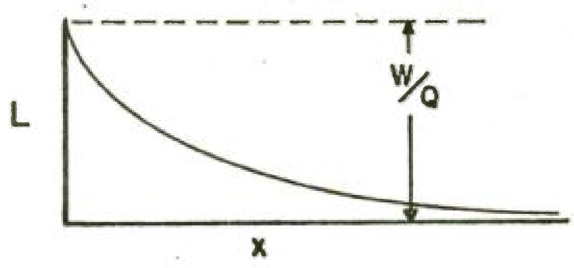
MATERIAL BALANCE: $Q\left[L(x) - L(x+\Delta x)\right] - K\bar{L}A\Delta x = 0$

DIFFERENTIAL EQUATION: $U\frac{dL}{dx} + KL = 0$

GENERAL SOLUTION: $L = (\text{CONSTANT})\, e^{-\frac{K}{U}x}$

BOUNDARY CONDITION: $L(0) = \frac{W}{Q}$

PARTICULAR SOLUTION: $L = \frac{W}{Q}\, e^{-\frac{K}{U}x}$



APPLICATIONS:

1. KNOWING W, Q, A, K; PREDICT L(x)
2. KNOWING L(x), Q, A; OBTAIN K

III. Basis of the Mathematical Analysis

This section considers the development of the one dimensional (variation along the longitudinal axis only) differential equation of mass transport in an estuary. The first part develops this equation by making a material balance over a small volume of the waterway, in which all parameters vary only with the longitudinal distance variable and time. The second part shows that this result is valid, provided the concentration of the substance transported is recognized to be the tidal-smoothed, area-averaged value. The final part summarizes several cases of estuarine mass transport and shows an example of the complete analysis through the development of the particular solution and a numerical application for steady state salinity profiles.

Development of the One Dimensional Equation

Transport of any substance in a tidal estuary is governed by the Law of Conservation of Mass. Figure 5 illustrates the application of this law for a non-conservative pollutant in an estuary. After discharge, waste particles are carried downstream toward the ocean by the movement of upland runoff. This phenomenon is known as convection. The rate of convective mass transport across any river section is equal to the product of fresh water runoff, Q, and contaminant concentration, L.

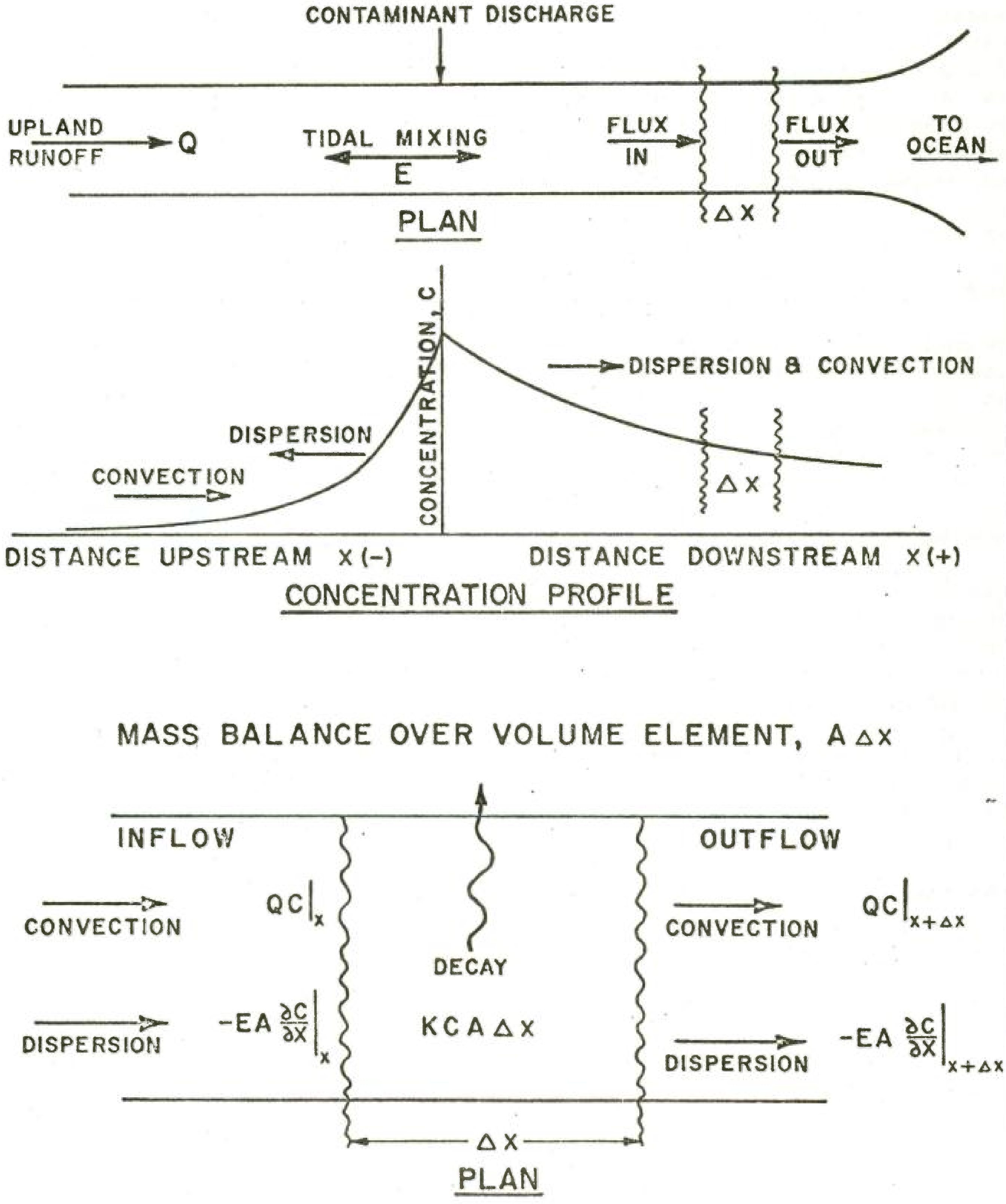
Particles are also transported in an estuary by longitudinal mixing. Mixing, or dispersion, is a complex function of reversing tidal currents and salinity-induced circulation patterns. Dispersive transport occurs only in the presence of a concentration gradient of the material being transported. The rate of dispersive transport is equal to the product of a dispersion coefficient, E, and the negative of the longitudinal concentration gradient, dL/dx. The dispersion coefficient, E, is a measure of the estuary's ability to transport material in the direction of a concentration gradient, regardless of the direction of net water movement.

The concentration profile in Figure 5 illustrates how convection and dispersion distribute estuarine contaminants. As only contaminants which decay are being considered, the maximum contaminant concentration from a particular discharge will occur at the point of introduction of that contaminant.

The concentration in the region downstream of the point of discharge, discharge at $x=0$, will decrease less rapidly than its upstream

FIGURE 5

CONTAMINANT DISTRIBUTION IN AN ESTUARY
MASS TRANSPORT RELATIONSHIPS



counterpart. This occurs because dispersion causes material to move in the direction of decreasing concentration, thus reinforcing convection. At the same distance from the point of introduction, the upstream concentration will be lower than the downstream value.

A material balance over the incremental volume, $A\Delta x$, in Figure 5 is written:

$$\text{INFLOW} - \text{OUTFLOW} + \text{PRODUCTION} = \text{ACCUMULATION} \quad \ldots\ldots (1)$$

The INPUT and OUTPUT terms are the sums of convective and dispersive transport across, respectively, the upstream and downstream faces of the volume element.

The PRODUCTION, or in this case, decay, term is the rate at which material is produced or consumed by the reaction process within the volume element. For example, the rate of biological destruction of organic matter in an estuary can be described by first order kinetics. The rate of BOD decay, therefore, is equal to the product of the unit rate, KL, times the volume, $A\Delta x$, within which the reaction is taking place. Decay of many other non-conservative contaminants, including coliforms and radioactivity and temperature, are also described by first order kinetics. For purposes of this report, first order kinetics are employed for all decay mechanisms.

The ACCUMULATION term completes the inventory by accounting for the increase or decrease of material upon summation of the rates of inflow, outflow and production. ACCUMULATION is equal to the time rate of change of total contaminant mass within the reactor volume, $A\Delta x$.

Algebraic summation of the individual terms shown in Figure 5 gives:

$$\left[QL - EA\frac{\partial L}{\partial x}\right]_x - \left[QL - EA\frac{\partial L}{\partial x}\right]_{x+\Delta x} - KLA\Delta x = \frac{\partial}{\partial t}[LA\Delta x] \quad \ldots\ldots (2)$$

Nomenclature used throughout this report are given on page 60. The notation $\left[QL-EA\frac{\partial L}{\partial x}\right]_x$ reads: "The quantity, $QL-EA\frac{\partial C}{\partial x}$, evaluated at the point x." The product KLA, rather than being evaluated at a single point such as x or $x+\Delta x$, represents that value which, when multiplied by Δx, yields the actual decay over the volume element. In the limit, as $\Delta x \rightarrow 0$, K, L and A will each be evaluated at the point x. Similarly, LA in the last term represents that value which, when multiplied by Δx, will yield the mass of pollutant contained within the volume element.

The parameters, Q, A, E and K, in most estuaries are functions of distance and time. To avoid mathematical complexity, these parameters are often considered to be constants. This approach was employed for the analysis used in this report. Justification of this procedure is given in Section IV, Parameter Evaluation.

For the case of constant Q, E, A and K, Equation 2 rearranges to:

$$E \left[\frac{\left(\frac{\partial L}{\partial x}\right)_{x+\Delta x} - \left(\frac{\partial L}{\partial x}\right)_{x}}{\Delta x} \right] - \frac{Q}{A} \left[\frac{(L)_{x+\Delta x} - (L)_{x}}{\Delta x} \right] - KL = \frac{\partial L}{\partial t} \quad \ldots\ldots\ldots (3)$$

The bracketed terms are average rates of change with respect to x. The limit of Equation 3, as Δx approaches zero, is:

$$E \frac{\partial^2 L}{\partial x^2} - U \frac{\partial L}{\partial x} - KL = \frac{\partial L}{\partial t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

U is equal to Q/A and is the average fresh water velocity. Equation 4 is a linear partial differential equation in x and t and is often referred to as the convection-diffusion equation for non-conservative substances. It has been selected as the defining equation for all subsequent analysis of estuarine BOD distribution presented in this report. A summary of the above development is given in Figure 6.

With respect to the defining equation for dissolved oxygen transport, in addition to the unit deoxygenation rate, KL, the rate of reoxygenation must be considered. The unit rate of reoxygenation is proportional to the dissolved oxygen deficit and may be written K_2D, in which D is the dissolved oxygen deficit, or the difference between oxygen saturation, C_s, and actual oxygen in the water, C. K_2 is the reaeration coefficient and represents the mechanism of oxygen transfer from the atmosphere to the estuary. Application of Equation 1 to oxygen transport follows the approach outlined above for BOD and yields, analogous to Equation 4:

$$E \frac{\partial^2 D}{\partial x^2} - U \frac{\partial D}{\partial x} - K_2 D + KL = \frac{\partial D}{\partial t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Equation 5 is the defining differential equation for the analysis of DO distribution presented herein. Notice that Equation 5 contains L, the BOD distribution. This demonstrates the cause and

FROM THE MATERIAL BALANCE TO THE DIFFERENTIAL EQUATION

$$\text{INFLOW} - \text{OUTFLOW} - \text{LOSSES} = 0$$

$$\left[QC - EA\frac{dc}{dx}\right]_x - \left[QC - EA\frac{dc}{dx}\right]_{x+\Delta x} - \overline{KCA}\,\Delta x = 0$$

DIVIDE BY $(-\Delta x)$ AND OBTAIN:

$$\frac{\left[QC - EA\frac{dc}{dx}\right]_{x+\Delta x} - \left[QC - EA\frac{dc}{dx}\right]_x}{\Delta x} + \overline{KCA} = 0$$

CALL $\left[QC - EA\frac{dc}{dx}\right]$ THE FUNCTION $f(x)$ AND REWRITE:

$$\frac{f(x+\Delta x) - f(x)}{\Delta x} + \overline{KCA} = 0$$

TAKE THE LIMIT AS $\Delta x \to 0$:

$$\text{LIMIT}_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \frac{df(x)}{dx} = \frac{d}{dx}\left[QC - EA\frac{dc}{dx}\right]$$

$$\text{LIMIT}_{\Delta x \to 0} \overline{KCA} = K\,C(x)\,A(x) = KCA$$

THE LIMIT OF THE EQUATION IS:

$$\frac{d}{dx}\left[QC - EA\frac{dc}{dx}\right] + KCA = 0$$

FOR THE CASE OF CONSTANT Q, A, E, THIS BECOMES :

$$E\frac{d^2c}{dx^2} - U\frac{dc}{dx} - KC = 0$$

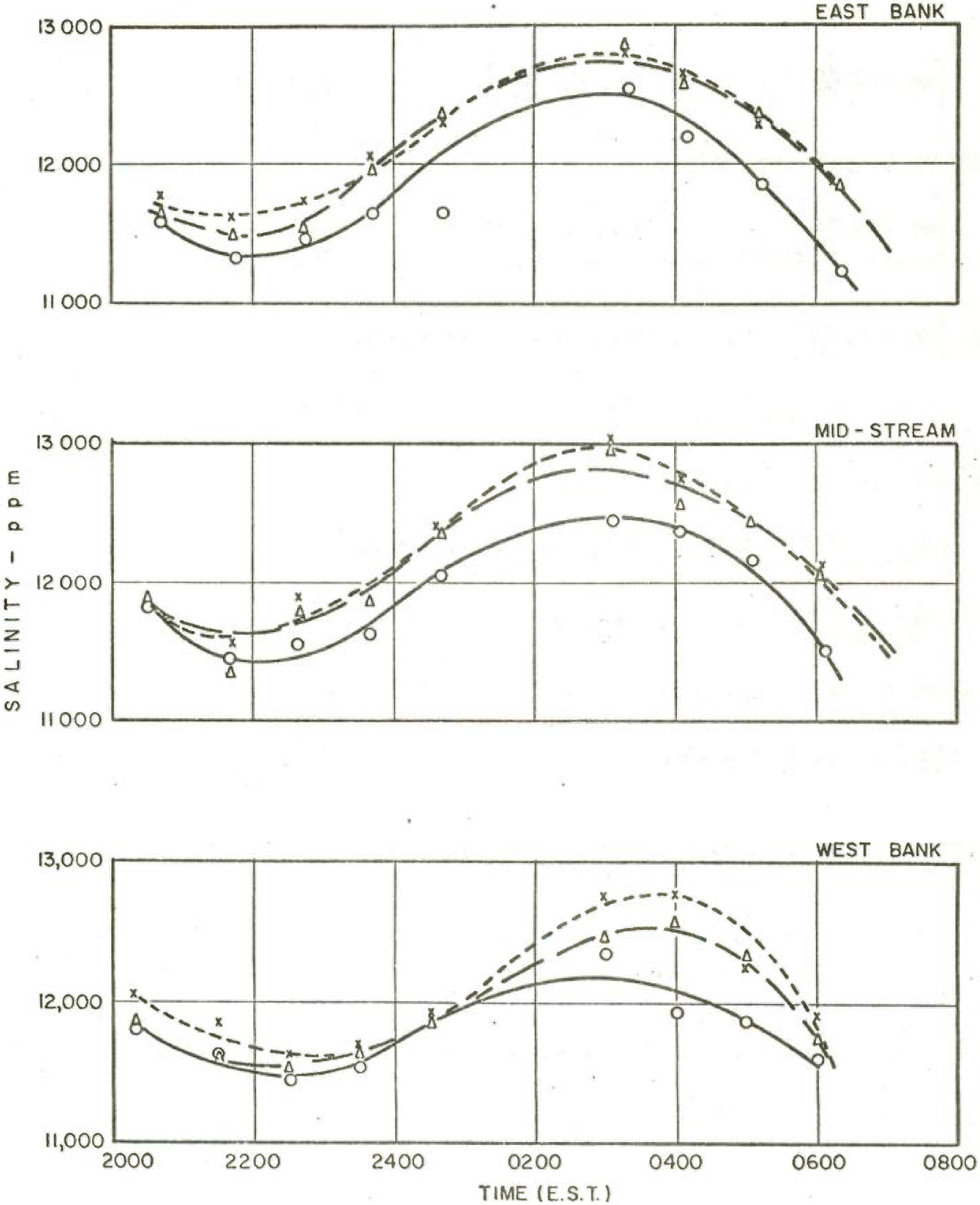
THIS IS THE DEFINING DIFERENTIAL EQUATION FOR STEADY STATE MASS TRANSPORT OF CONTAMINANTS SUBJECT TO FIRST ORDER DECAY.

FIGURE 7

STATION - DOBBS FERRY
SALINITY MEASUREMENTS
LOCATION - LAT. 41°-01'
MILE POINT 22.825
DATE NOV. 23-24, 1964

DEPTH IN FT.

	E. BANK	MID. S.	W. BANK
O	1	1	1
Δ	19	25	12.5
x	27.5	42	21



effect relationship between BOD and DO in an estuary. Prior to integration of Equation 5, Equation 4 must be integrated to obtain the explicit behavior of L as a function of x and t. This function is then substituted in Equation 5 and the second integration performed.

It is important to note that the concentration of BOD and DO used is for a tidal-smoothed and area-averaged concentration. No attempt has been made to define conditions for a given point within the cross-section or over the tidal cycle.

Reduction to the One Dimensional Form

To establish the tidal-smoothed, area-averaged nature of the pollutant concentrations described above, and to understand clearly the applicability of the one dimensional equation, start with the equation of continuity of a single chemical specie (3), in which contaminant concentration is a function of three space dimensions and real time. Dependence on the lateral and vertical space coordinates is replaced by dependence on total cross-sectional area by integrating over the total width and depth. The resulting equation is then integrated over a tidal cycle and change with respect to real time replaced by change with respect to tidal cycle units of time.

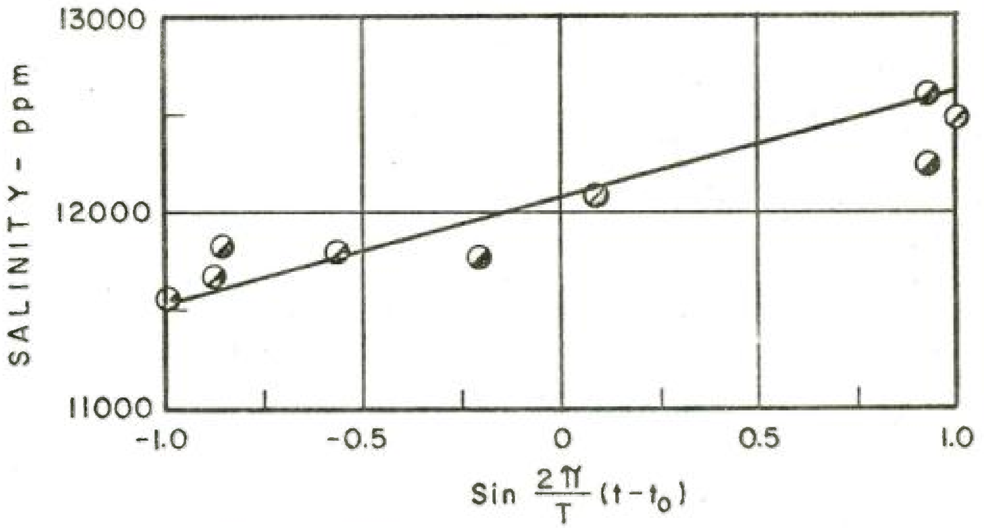
In the course of these integrations, several new terms are generated, all of which contribute to the dispersion phenomenon. These are eventually replaced by the overall dispersion flux, $E\frac{dc}{dx}$.

Figures 7 through 10 illustrate the relation between point values and the time-smoothed, area-averaged value for the case of Hudson River salinity. The actual variation of salinity across two typical cross-sections within the salt intruded reach of the River is shown on Figures 7 and 9. Figures 8 and 10 show the sinusoidal variation of the area-averaged salinity at these sections over a tidal cycle, a linearized plot of this variation, and the average or tidal-smoothed value of these area-averaged values.

The time-smoothed equation of continuity of a single reactant in a turbulent fluid mixture is discussed by Bird (3) and can be written:

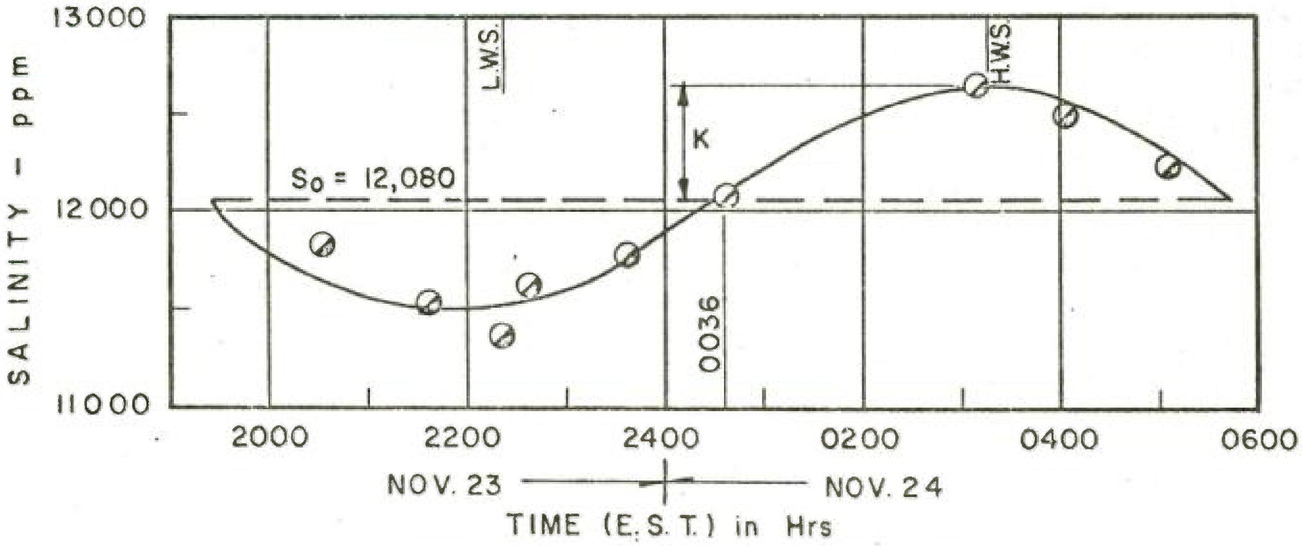
$$\frac{\partial c}{\partial t} + \overline{V} \cdot \vec{\eta} + r = 0 \qquad (6)$$

STATION - DOBBS FERRY
LOCATION - LAT. 41°-01'
MILE POINT 22.825
DATE NOV. 23-24, 1964



S_0 = 12,080 ppm
t_0 @ 0036 Hrs
K = 550 ppm

MEAN SECTIONAL SALINITY VS. PHASE ANGLE



MEAN SECTIONAL SALINITY VS. TIME

FIGURE 9

STATION - POUGHKEEPSIE
SALINITY MEASUREMENTS
LOCATION - LAT. 41°-42.5'
MILE POINT 75.380
DATE NOV. 20, 1964

DEPTH IN FT.

	E. BANK	MID. S.	W. BANK
O	1.5	1.5	1.5
Δ	25	28	31.5
x	42	44.5	51

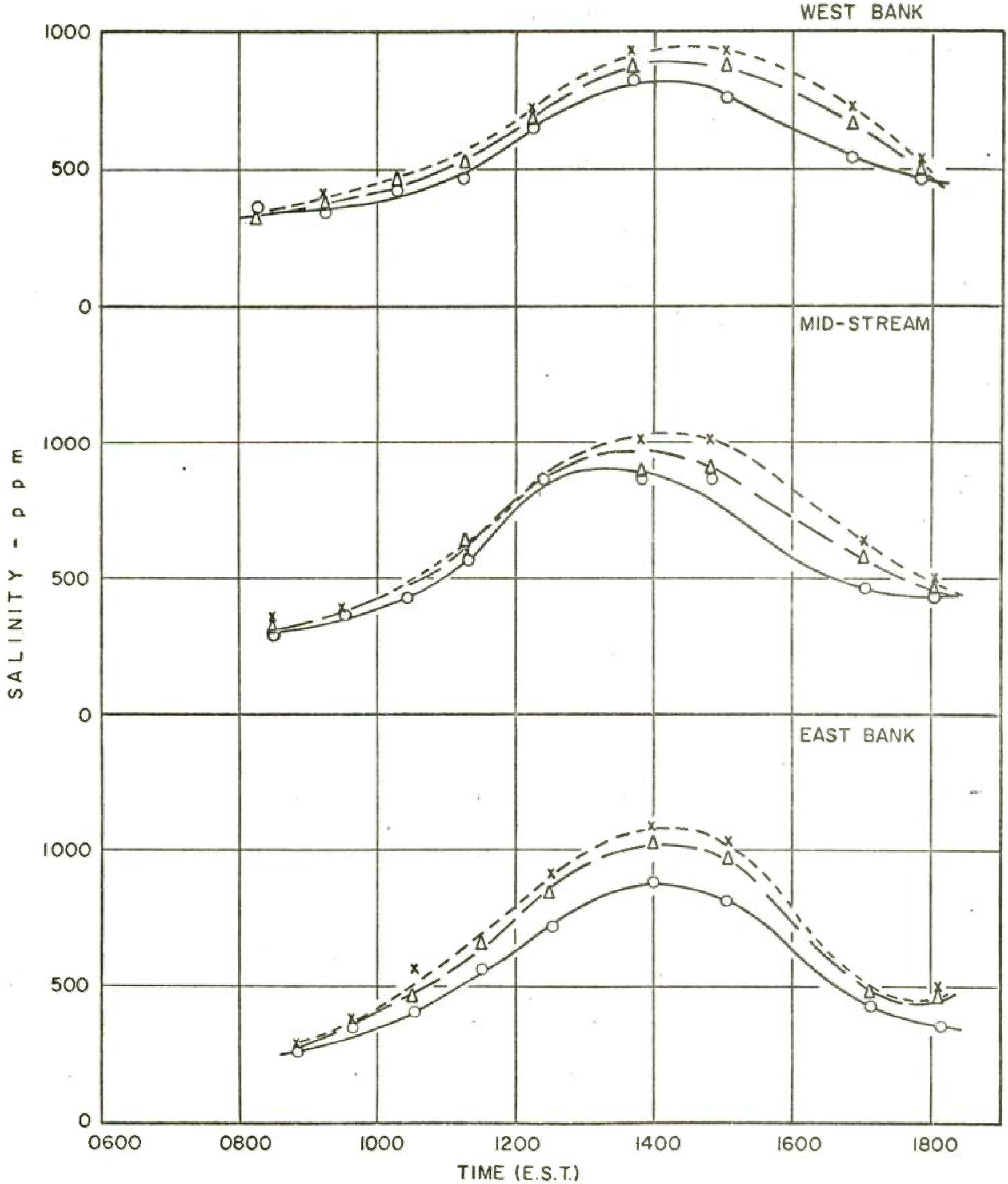
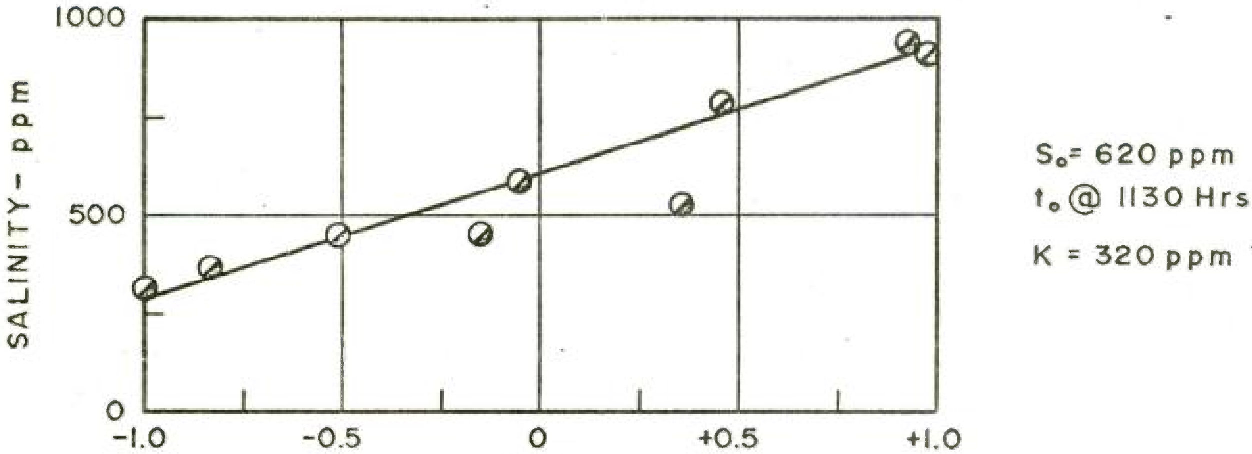


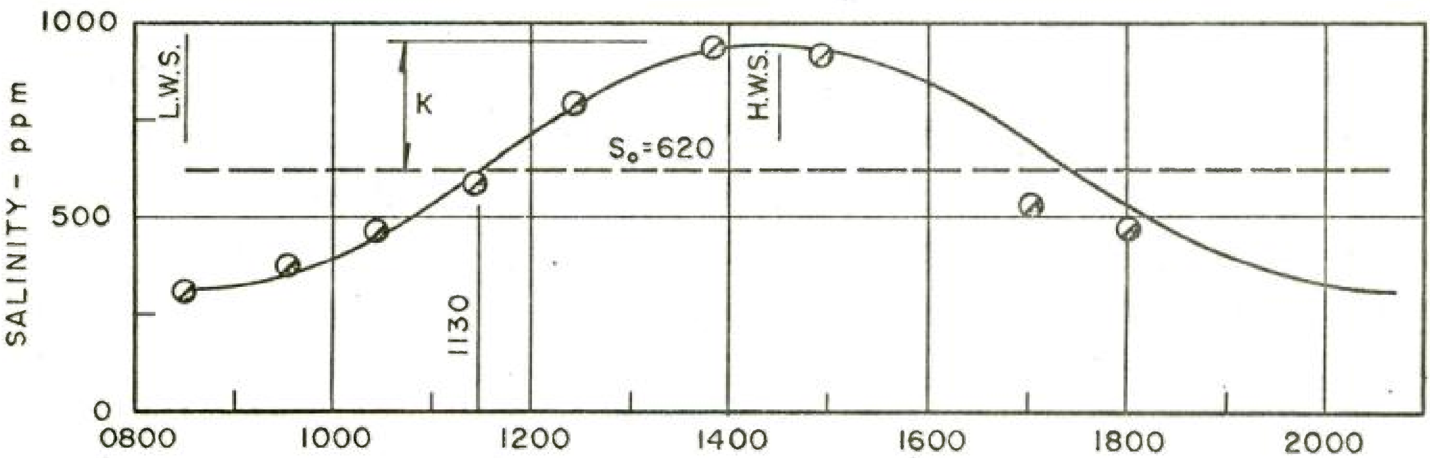
FIGURE 10

STATION - POUGHKEEPSIE
LOCATION - LAT. 41°-42.5'.
MILE POINT - 75.380
DATE - NOV. 20, 1964



$\sin \frac{2\pi}{T}(t-t_o)$

MEAN SECTIONAL SALINITY VS. PHASE ANGLE



TIME (E.S.T.) in Hrs.
MEAN SECTIONAL SALINITY VS. TIME

The one dimensional form of Equation 6 is obtained by integrating Equation 6 over a cross-section, A(x). In Cartesian form this is:

$$\int_{z_1(x)}^{z_2(x)} \int_{y_1(x,z)}^{y_2(x,z)} \left\{\frac{\partial c}{\partial t} + \frac{\partial \eta x}{\partial x} + \frac{\partial \eta y}{\partial y} + \frac{\partial \eta z}{\partial z} + r\right\} \partial y \partial z = 0 \ldots\ldots\ldots\ldots (7)$$

Use of the Leibnitz rule for the differentiation of an integral, the fundamental theorem for partial derivatives, the definition of an average and the fact that there is no net movement through the walls of the channel gives, for a constant area A:

$$\frac{\partial \langle c \rangle}{\partial t} + \frac{\partial \langle \eta_x \rangle}{\partial \eta} + \langle r \rangle = 0 \ldots\ldots\ldots\ldots (8)$$

In terms of bulk transport and diffusion, $\langle \eta_x \rangle$ is given by $[\langle cV_x \rangle + \langle j_x \rangle]$. Contaminant decay can normally be described by first order kinetics $\langle r \rangle = K\langle c \rangle$.

Equation 8 becomes:

$$\frac{\partial \langle c \rangle}{\partial t} + \frac{\partial}{\partial x} [\langle cV_x \rangle + \langle j_x \rangle] + K\langle c \rangle = 0 \ldots\ldots\ldots\ldots (9)$$

$\langle cV_x \rangle$, the average of the product of $C(x,y,z,t)$ and $V_x(x,y,z,t)$ across any cross-section, cannot be obtained without detailed knowledge of the velocity and concentration gradients over a cross-section.

To avoid this difficulty, it is customary to replace $\langle cV_x \rangle$ by its equivalent $\alpha\langle V_x \rangle\langle c \rangle$. This procedure merely defines a function, $\alpha(x,t)$, as the ratio of the area average of the product of the contaminant concentration times the longitudinal velocity to the product of the area averages of these two variables. Substitution in Equation 9 and slight rearrangement yields:

$$\frac{\partial \langle c \rangle}{\partial t} + \frac{\partial}{\partial x}(\langle V_x \rangle\langle c \rangle) + \frac{\partial}{\partial x} [(\alpha-1)\langle V_x \rangle\langle c \rangle+\langle j_x \rangle] + K\langle c \rangle = 0 \ldots\ldots (10)$$

The term $\frac{\partial}{\partial x}(\langle V_x \rangle\langle c \rangle)$ represents the rate of change in the longitudinal direction of the bulk transport flux, $(\langle V_x \rangle\langle c \rangle)$, i.e., the transport of material by virtue of the mean velocity of the estuary. The next term represents the rate of change of the longitudinal dispersion or the additional transport of material over that by mean motion. This term indicates that longitudinal dispersion consists of turbulent and molecular diffusion in the x direction, $\langle j_x \rangle$, and of bulk motion when both velocity and concentration vary across a cross-section. It is customary to represent each of these mechanisms by the product of a longitudinal mixing parameter and the longitudinal concentration gradient. For example, j_x is often $-(\varepsilon + \mathcal{D}_{AB})\frac{\partial c}{\partial x}$ in which ε is the turbulent or eddy diffusivity and $\mathcal{D}_{AB}$ is the molecular diffusivity of the mixture. Therefore, replace $[(\alpha-1)\langle V_x\rangle\langle c\rangle+\langle j_x\rangle]$ by $-E\frac{\partial\langle c\rangle}{\partial x}$ in which E is known as the longitudinal dispersion coefficient. Equation 10 becomes:

$$\frac{\partial}{\partial x}\left(E\frac{\partial c}{\partial x}\right) - \frac{\partial}{\partial x}(\langle V_x\rangle\langle c\rangle) - K\langle c\rangle = \frac{\partial\langle c\rangle}{\partial t}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(11)$$

In a tidal estuary, $\langle V_x\rangle$, by virtue of the tidal motion, is a function of both time and space. Furthermore, a description of the behavior of $\langle c\rangle$ at every point within a tidal cycle is often unnecessary; i.e., the average behavior over a tidal cycle or perhaps behavior at significant tidal modes, such as at high or low water or at slack, is often sufficient.

To obtain a description of average behavior, Equation 9 is integrated with respect to time over a tidal cycle and then divided by the tidal period, T. This time averaging is identical to the procedure used to obtain the time-smoothed Navier-Stokes equations of turbulent flow. The resulting equation is:

$$\frac{\partial\langle\bar{c}\rangle}{\partial t} + \frac{\partial}{\partial x}(\langle\overline{cV_x}\rangle + \langle\bar{j}_x\rangle) + K\langle\bar{c}\rangle = 0\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(12)$$

in which:

$$\langle\overline{cV_x}\rangle = \frac{1}{T}\int_t^{t+T}\langle cV_x\rangle\partial t = \frac{1}{T}\int_t^{t+T}\alpha\langle c\rangle\langle V_x\rangle\partial t$$

$\langle\overline{cV_x}\rangle$ can be rewritten in terms of products of averages instead of the average of a product. Just as in the case of time-smoothing for turbulence, additional terms accrue since the above integral contains products of terms each of which are time variable. Let $\alpha=\bar{\alpha}+\alpha'$, $\langle c\rangle=\langle\bar{c}\rangle+\langle c\rangle'$ and $\langle V_x\rangle=\langle\bar{V}_x\rangle+\langle V_x\rangle'$ and obtain:

$$\frac{\partial\langle\bar{c}\rangle}{\partial t} + \frac{\partial}{\partial x}[\bar{\alpha}\langle\bar{c}\rangle\langle\bar{V}_x\rangle + F + \langle\bar{j}_x\rangle] + K\langle\bar{c}\rangle = 0\ldots\ldots\ldots\ldots\ldots\ldots(13)$$

in which:

$$F = \langle\bar{V}_x\rangle\overline{\alpha'\langle c\rangle'} + \langle\bar{c}\rangle\overline{\alpha'\langle V_x\rangle'} + \bar{\alpha}\overline{\langle c\rangle'\langle V_x\rangle'} + \overline{\alpha'\langle c\rangle'\langle V_x\rangle'}$$

Employing the same procedure used to obtain Equation 11 from Equation 10, Equation 13 becomes:

$$\frac{\partial}{\partial x}\left(E\,\frac{\partial\langle\bar{c}\rangle}{\partial x}\right) - \frac{\partial}{\partial x}\left(\langle\bar{V}_x\rangle\langle\bar{c}\rangle\right) - K\langle\bar{c}\rangle = \frac{\partial\langle\bar{c}\rangle}{\partial t} \quad \ldots\ldots\ldots\ldots\ldots\ldots (14)$$

in which:

$$-E\,\frac{\partial\langle\bar{c}\rangle}{\partial x} = (\bar{\alpha}-1)\langle\bar{c}\rangle\langle\bar{V}_x\rangle + F + \langle\bar{j}_x\rangle$$

Therefore, if the description of estuarine mass transport is made in terms of area and tidal cycle averages, the longitudinal dispersion flux, $\left(-E\,\frac{\partial\langle\bar{c}\rangle}{\partial x}\right)$ consists of contributions due to tidal fluctuations, F, the presence of lateral and vertical velocity and concentration profiles, $(\bar{\alpha}-1)\langle\bar{c}\rangle\langle\bar{V}_x\rangle$, and turbulent and molecular diffusion, $\langle\bar{j}_x\rangle$. $\langle\bar{j}_x\rangle$ is always negligible in comparison to both former mechanisms. In the region of high salinity close to the mouth of the estuary, values of E have been shown to be markedly higher than in the upstream reaches of the estuary where salinities are low. This is ascribed to vertical and longitudinal density differences which are associated with salinity gradients and which cause significant circulation. In terms of the above development, this phenomenon means large values of $\bar{\alpha}$ due to marked changes in concentration and velocity across a cross-section. In the upper reaches of the tidal estuary, $\bar{\alpha}$ is of a lower order of magnitude and the terms contained in F are probably the major contributors to E.

$\langle\bar{V}_x\rangle$, the tidal-smoothed, area-averaged velocity is just the average fresh water velocity, U, since tidal motion contributes no net transport of the carriage medium. Taking the usual working condition of fresh water flow independent of x and redefining $\langle\bar{c}\rangle$ as c, the working equation for channels of constant cross-section is:

$$\frac{\partial}{\partial x}\left(E\,\frac{\partial c}{\partial x}\right) - U\,\frac{\partial c}{\partial x} - Kc = \frac{\partial c}{\partial t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

For constant E, Equation 15 reduces to Equation 4. Since c is to be interpreted as the area-averaged, tidal-smoothed concentration, were Equation 15 to be used with measured concentrations to obtain numerical values of the parameters, it would appear necessary to obtain values of c at each station over a whole tidal cycle. However, assuming that the ratio of the concentration at any fixed point on the tidal cycle to the tidal-smoothed average concentration is independent of x and t, i.e., the station and tidal cycle, the tidal-smoothed concentration can be replaced by the concentration at said fixed point. This condition is the basis of the "same slack technique" of sampling, commonly employed in laboratory experiments and field surveys in which measurements are made at high and low water slack only. Actually, if this condition of a fixed ratio does not exist, slack measurements may still be used; the definition of E merely changes to include the required correction.

For estuaries of variable cross-section, A(x), Equation 7 yields:

$$\frac{\partial \langle c \rangle}{\partial t} + \frac{1}{A}\frac{\partial}{\partial x}\left(A\langle \eta_x \rangle\right) + K\langle c \rangle + G = 0 \quad \ldots\ldots (16)$$

In Equation 16, term G represents fluxing along the boundary stream lines and can be shown to be entirely negligible in the usual cases of interest. Replacing the flux $\langle \eta_x \rangle$ by its equivalent $\left[Uc - E\frac{\partial c}{\partial x}\right]$, Equation 16 becomes:

$$\frac{1}{A}\frac{\partial}{\partial x}\left(EA\frac{\partial c}{\partial x}\right) - \frac{1}{A}\frac{\partial}{\partial x}(UAc) - Kc = \frac{\partial c}{\partial t} \quad \ldots\ldots (17)$$

Had the parameters E, A, Q and K not been chosen constant, the limit of Equation 2, as x approached zero, would have been identical to Equation 17.

A summary of the above analysis is shown in Figure 11.

Typical Cases of Estuarine Mass Transport

Figure 12 summarizes the defining differential equations for several cases of estuarine mass transport. The last five equations are each special cases of the first equation, which is the general form of the one dimensional equation. In each case, the conditions specified permit the simplification.

For example, in the case of steady state salt transport with constant system parameters, there are no sources or sinks, since salt is a conservative substance and enters the estuary at its boundary. Steady state means that there is no time dependence, so the time derivative is zero. Since the parameters E and U are constant, they do not depend on x and may be taken outside of the differential operator, $\partial/\partial x$.

Figure 13 shows the application of the overall mathematical analysis to the development of steady state salinity profiles. After obtaining the working form of the differential equation for the

INTERPRETATION OF ONE-DIMENSIONAL ANALYSIS

THREE DIMENSIONAL MASS TRANSPORT EQUATION:

$$\frac{\partial c}{\partial t} + \frac{\partial uc}{\partial x} + \frac{\partial vc}{\partial y} + \frac{\partial wc}{\partial z} + D + R = 0$$

INTEGRATE OVER CROSS-SECTIONAL AREA:

$$\frac{\partial \langle c \rangle}{\partial t} + \frac{1}{A}\frac{\partial A\langle uc \rangle}{\partial x} + \langle R \rangle + G = 0$$

REPLACE $\langle c \rangle$ BY C, $\langle uc \rangle$ BY $\left[UC + \overline{u'c'}\right]$ AND OBTAIN:

$$\frac{\partial C}{\partial t} + \frac{1}{A}\frac{\partial}{\partial x}(AUC) + \langle R \rangle + \frac{1}{A}\frac{\partial}{\partial x}(A\overline{u'c'}) = 0$$

REPLACE $\overline{u'c'}$ BY $-E_T\frac{\partial C}{\partial x}$, AU BY Q_T AND OBTAIN:

$$\frac{1}{A}\frac{\partial}{\partial x}\left[E_T A\frac{\partial C}{\partial x} - Q_T C\right] - \langle R \rangle = \frac{\partial C}{\partial t}$$

INTEGRATE OVER TIDAL CYCLE. LET $\int_0^T Q_T C\,dt = Q\bar{C} + \overline{Q_T' C'}$.

LUMP ALL TRANSPORT ITEMS OTHER THAN $Q\bar{C}$ INTO DISPERSION FLUX AND OBTAIN:

$$\frac{1}{A}\frac{\partial}{\partial x}\left[EA\frac{\partial \bar{C}}{\partial x} - Q\bar{C}\right] - \langle R \rangle = \frac{\partial \bar{C}}{\partial t}$$

DIFFERENTIAL EQUATIONS DESCRIPTIVE OF ESTUARINE MASS TRANSPORT

1. GENERAL, ONE DIMENSIONAL, UNSTEADY STATE EQUATION

$$\frac{1}{A}\frac{\partial}{\partial x}\left[EA\frac{\partial C}{\partial x} - QC\right] + \text{SOURCES} - \text{SINKS} = \frac{\partial C}{\partial t}$$

2. STEADY STATE TRANSPORT OF OCEAN SALT INTO AN ESTUARY

$$\frac{1}{A}\frac{d}{dx}\left[EA\frac{dC}{dx} - QC\right] = 0$$

3. STEADY STATE SALT TRANSPORT, CONSTANT SYSTEM PARAMETERS

$$E\frac{d^2C}{dx^2} - U\frac{dC}{dx} = 0$$

4. INSTANTANEOUS RELEASE OF NON-CONSERVATIVE DYE, CONSTANT PARAMETERS

$$E\frac{\partial^2 C}{\partial x^2} - U\frac{\partial C}{\partial x} - KC = \frac{\partial C}{\partial t}$$

5. CONTINUOUS DISCHARGE OF LINE SOURCE OF ORGANIC WASTE, STEADY STATE

$$\frac{1}{A}\frac{d}{dx}\left[EA\frac{dC}{dx} - QC\right] + \frac{w(x)}{A} - KC = 0$$

6. DISSOLVED OXYGEN TRANSPORT, SINGLE PLANE SOURCE OF BOD, STEADY STATE

$$E\frac{d^2C}{dx^2} - U\frac{dC}{dx} + K_2(C_s - C) - KL(x) = 0$$

DEVELOPMENT OF STEADY STATE SALINITY PROFILES

GENERAL EQUATION: $\frac{1}{A}\frac{d}{dx}\left[EA\frac{dC}{dx} - QC\right] + \langle R\rangle = 0$

CONDITIONS: NO SOURCES OR SINKS

WORKING DIFFERENTIAL EQUATION: $\frac{d}{dx}\left[EA\frac{dC}{dx} - QC\right] = 0$

INTEGRATE ONCE TO OBTAIN: $E\frac{dC}{dx} - \frac{Q}{A}C = C_1$ (INTEGRATION CONSTANT)

1ST BOUNDARY CONDITION: $\left[E\frac{dC}{dx} - UC\right]_{x=0} = 0$

FIT GENERAL SOLUTION TO BC: $C_1 = 0$

REARRANGE EQUATION: $\frac{dC}{C} = \frac{U}{E}\,dx$

INTEGRATE AGAIN; INCORPORATE 2nd BOUNDARY CONDITION INTO LOWER LIMIT: $\int_{C_0}^{C}\frac{dC}{C} = \int_{0}^{X}\frac{U}{E}\,dx$

PARTICULAR SOLUTION, VARIABLE PARAMETERS: $C = C_0 e^{\int_0^X \frac{U}{E}dx}$

PARTICULAR SOLUTION, CONSTANT PARAMETERS: $C = C_0 e^{U/E\,X}$

conditions specified, two integrations, each accompanied by application of a suitable boundary condition, are performed to yield the particular solution.

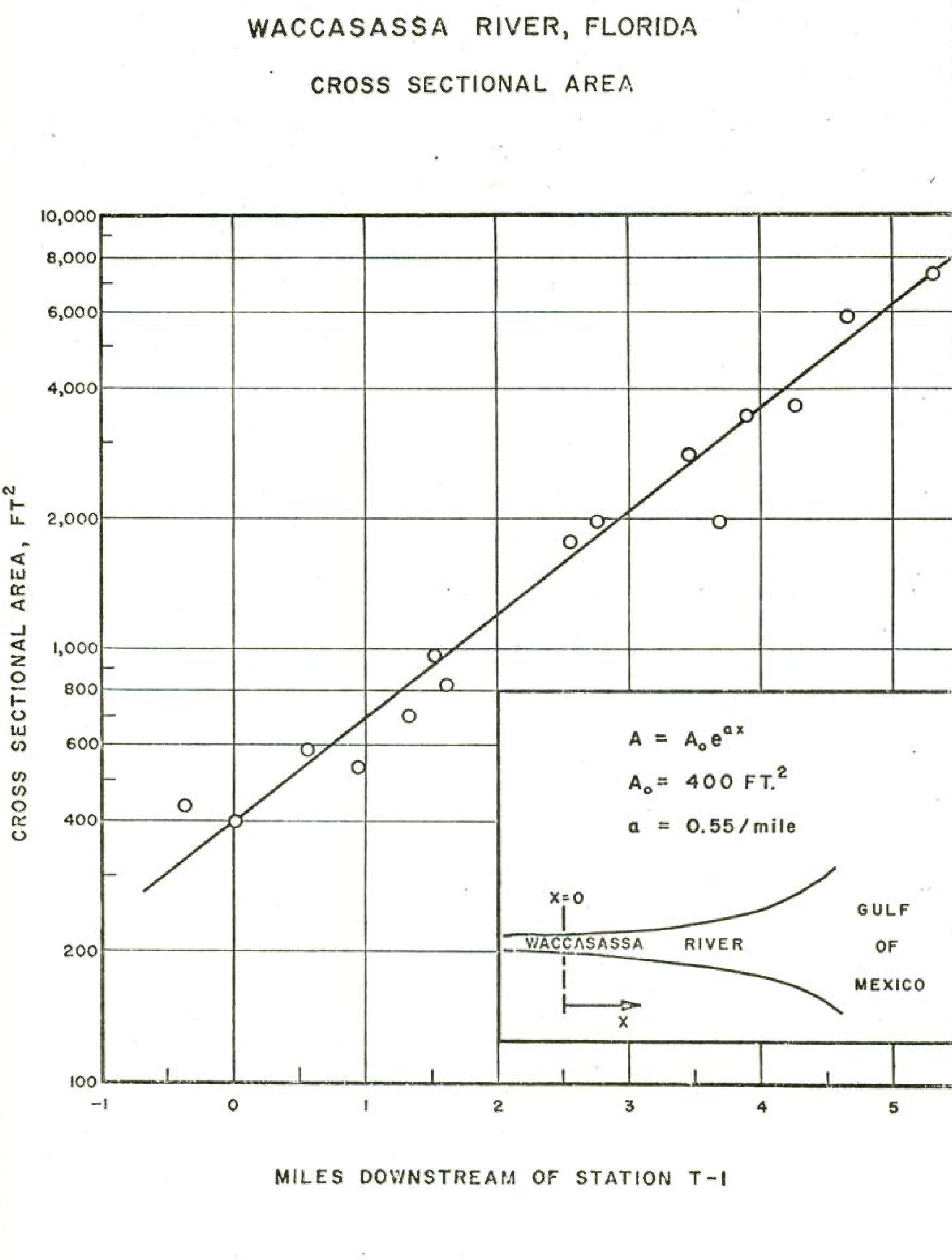
Figures 14, 15 and 16 portray the application of the steady state salinity profiles, obtained in Figure 13, to the determination of the longitudinal dispersion coefficient, E. Although the example considers an estuary well outside the subject area of this study, it was selected because it clearly demonstrates the power of the analytical approach, when used correctly.

Early analysis (4) of the salinity profiles in this estuary assumed that the system parameters Q, A and E were constant. The final equation on Figure 13 was used but the data fit this equation poorly.

Careful study of the estuary showed that the cross-sectional area of the estuary was not constant but varied exponentially, as shown in Figure 14. A second analysis (5) recognized this variable behavior and employed the variable parameter salt equation shown in Figure 13.

Results for the case of variable area and a constant flow are shown in Figure 15. Results for the case of the exponentially varying area, accompanied by a linearly varying flow are shown in Figure 16. In both cases, the dispersion coefficient was assumed to be constant; the good fit of the data to the functional form of the integrated equations shows the assumption was correct.

Notice the results varied little, whether constant and variable flow was assumed, whereas the two studies (4), (5) showed marked differences for constant versus variable area. This is explained by the fact that, over the reach studied, a twenty-fold variation in area occurred, while the flow variation over the same stretch was only 50% (100 to 150 cfs).

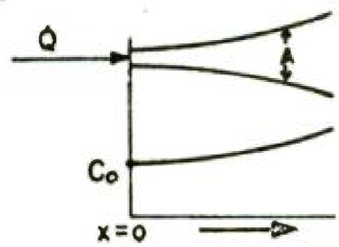

WACCASASSA RIVER, FLORIDA
CROSS SECTIONAL AREA
10,000
8,000
6,000
4,000
2,000
1,000
800
600
400
200
100
CROSS SECTIONAL AREA, FT²
-1
0
1
2
3
4
5
MILES DOWNSTREAM OF STATION T-1
A = A₀e^{ax}
A₀= 400 FT.²
a = 0.55/mile
X=0
WACCASASSA
RIVER
GULF
OF
MEXICO
X

WACCASASSA RIVER, FLORIDA

DETERMINATION OF DISPERSION COEFFICENT

CASE I – VARIABLE AREA, CONSTANT FLOW

$$C = C_o \, e^{\int_o^x \frac{U}{E} dx}$$



$$A = A_o \, e^{ax}$$

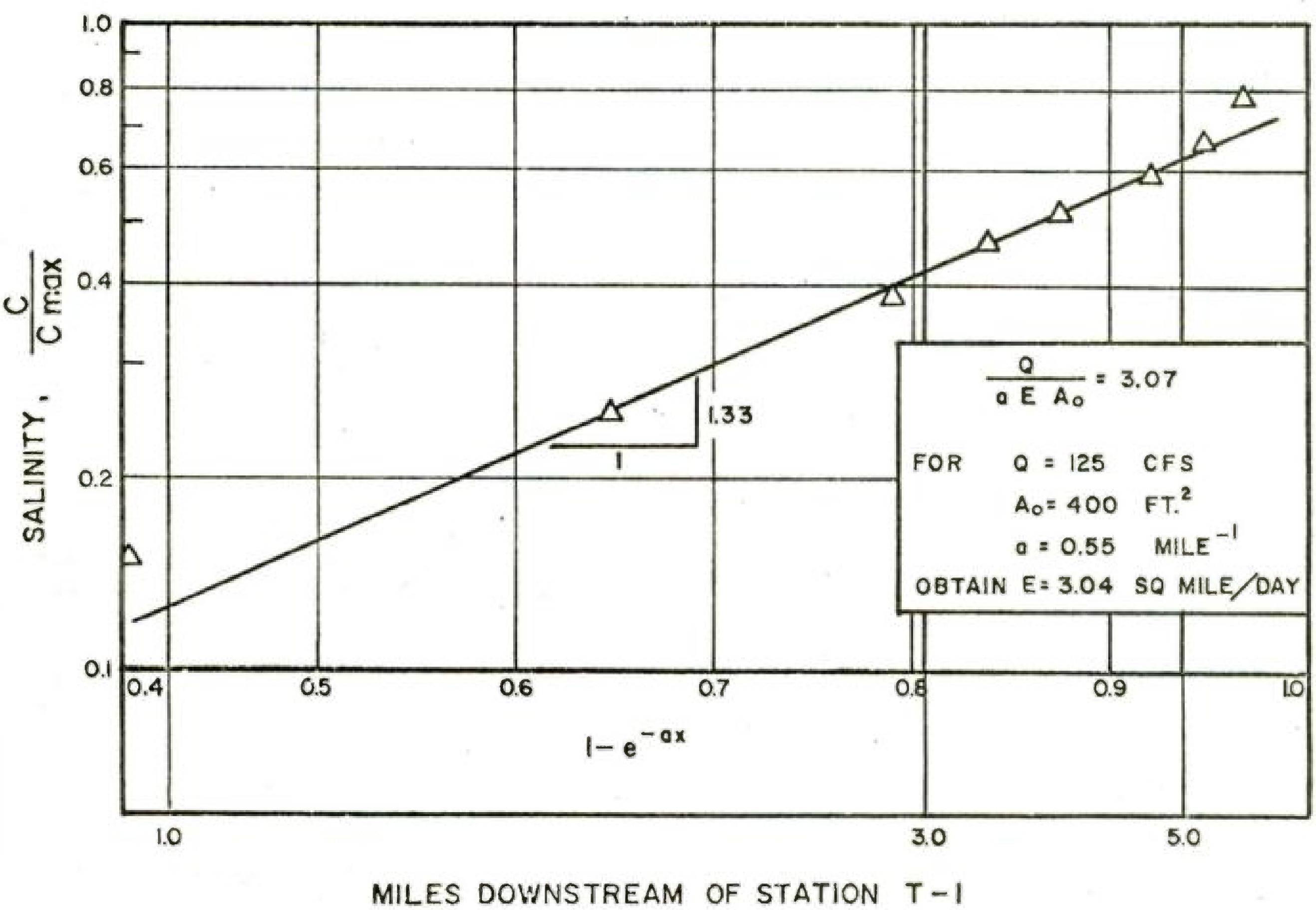
$$U = \frac{Q}{A_o} \, e^{-ax}$$

$$C = C_o \, e^{\int_o^x \frac{Q}{A_o E} e^{-ax} dx}$$

$$C = C_o \, e^{\frac{Q}{a A_o E}\left[1 - e^{-ax}\right]}$$

$$\ln \frac{C}{C_o} = \frac{Q}{a A_o E}\left[1 - e^{-ax}\right]$$

$$\log \frac{C}{C\,max} = \log \frac{C_o}{C\,max} + \left(\frac{1}{2.303}\right)\frac{Q}{a A_o E}\left[1 - e^{-ax}\right]$$

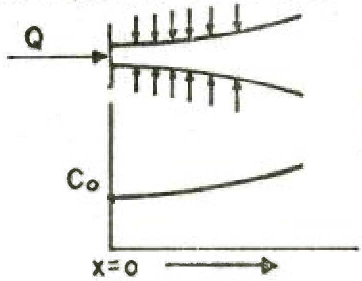


WACCASASSA RIVER, FLORIDA

DETERMINATION OF DISPERSION COEFFICIENT

CASE II - AREA AND FLOW BOTH VARIABLE

$$C = C_o \, e^{\int_0^x \frac{U}{E} dx}$$



$$Q = Q_o(1 + qx)$$

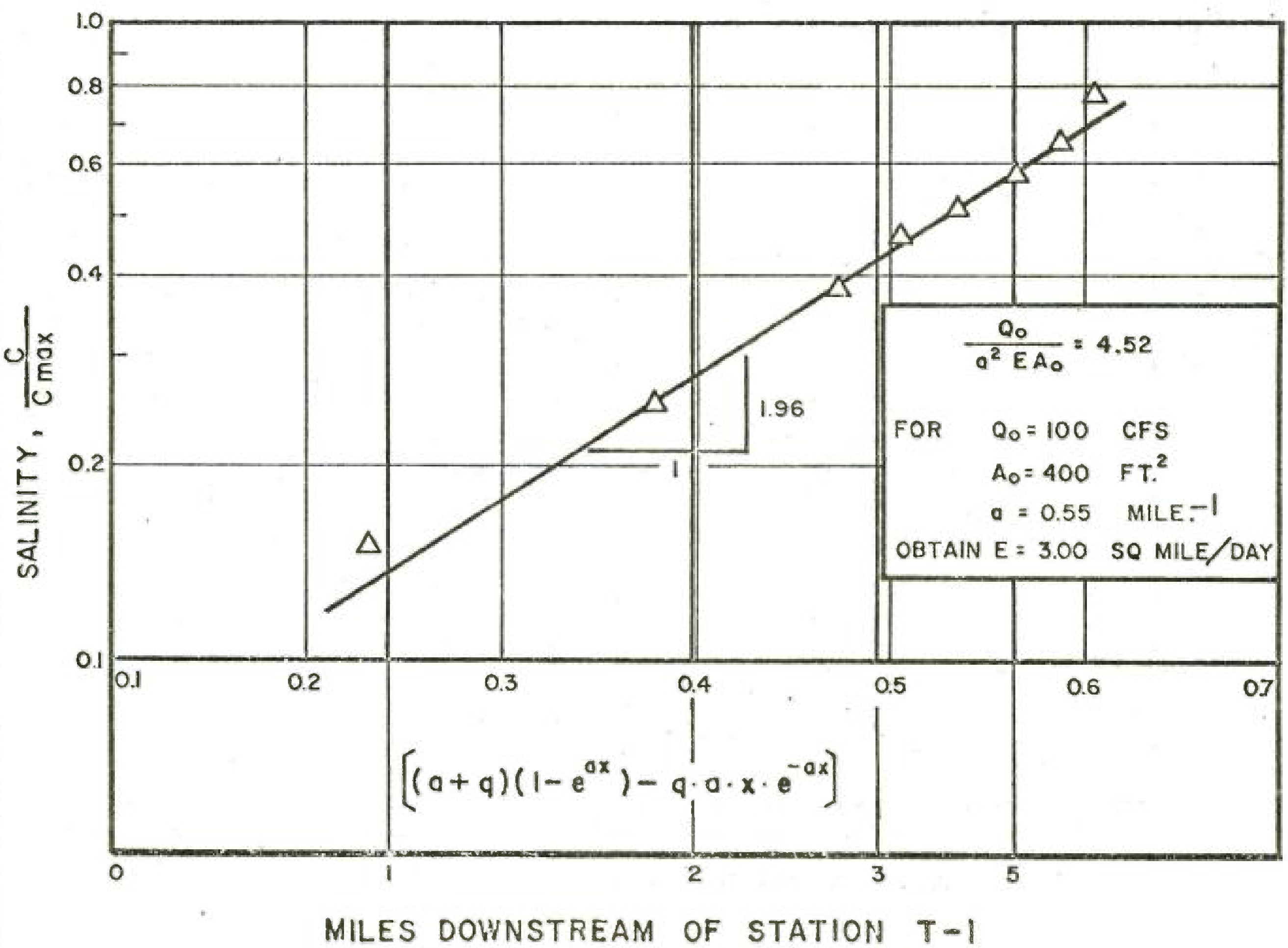
$$A = A_o \, e^{ax}$$

$$U = \frac{Q_o}{A_o}(1 + qx)\, e^{-ax}$$

$$C = C_o \, e^{\int_0^x \frac{Q_o}{EA_o}(1 + qx) e^{-ax} dx}$$

$$\ln \frac{C}{C_o} = \frac{Q_o}{a^2 E A_o}\left[(a+q)(1-e^{-ax}) - qaxe^{-ax}\right]$$

$$\mathrm{Log}\, \frac{C}{C\ max} = \mathrm{Log}\, \frac{C_o}{C\ max} + \left(\frac{1}{2.3}\right)\frac{Q_o}{a^2 E A_o}\left[(a+q)(1-e^{-ax}) - q \cdot a \cdot x \cdot e^{-ax}\right]$$



IV. Evaluation of Parameters and Waste Loads

Numerical values of the system parameters Q, A, U, E, K, K_a, R, V, C_s, which appear in the defining differential equations and boundary conditions and, therefore, control the distribution of any contaminant in the estuary, must be chosen for the Hudson, East, Hackensack, Passaic and Raritan Rivers, the Arthur Kill and Upper New York, Newark and Raritan Bays. Brief descriptions of the methods used to obtain these parameters and selections of the actual numerical values are given below. Waste loads, in #/day of BOD, are also given.

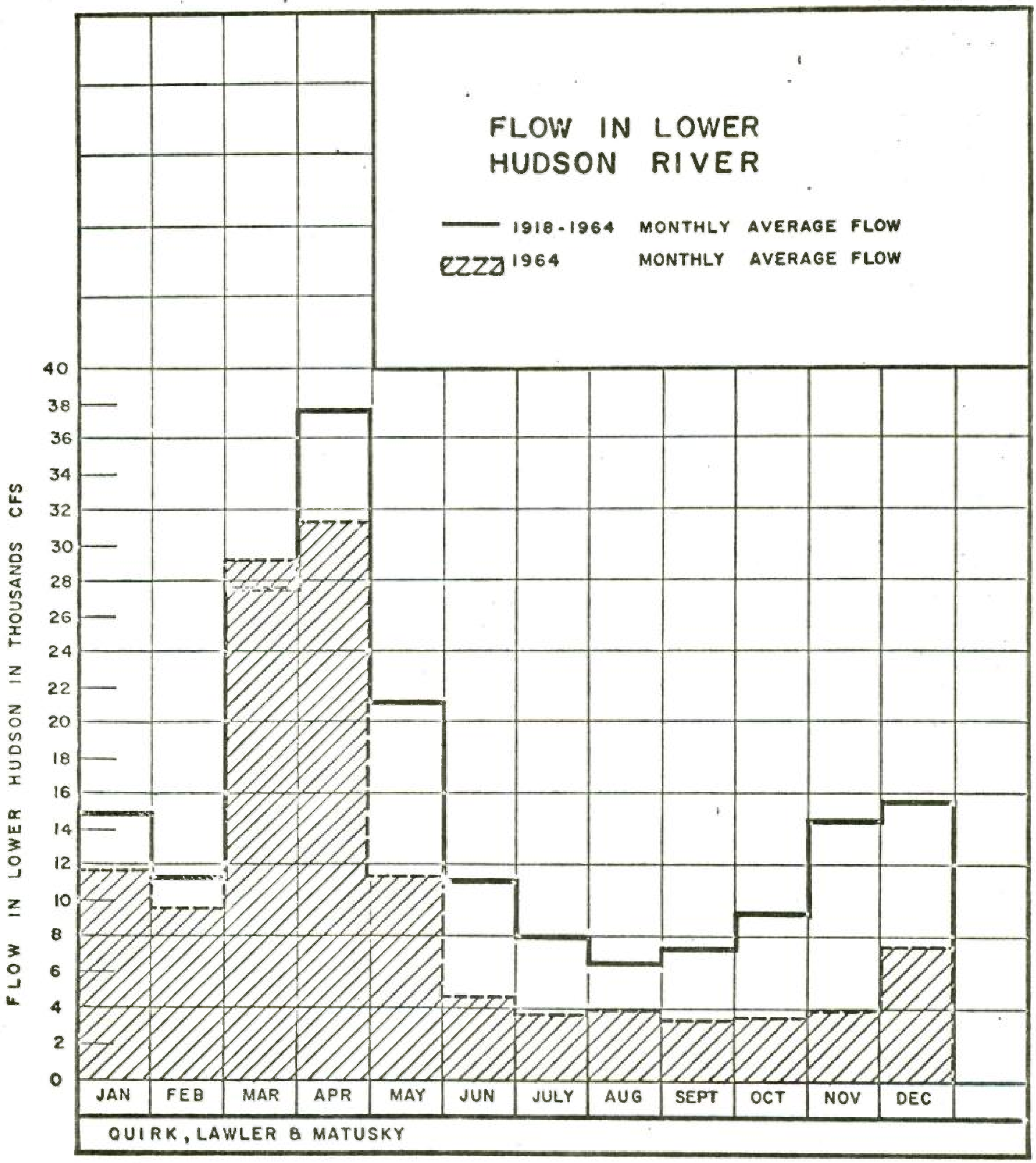
Fresh Water Flow and Velocity

Fresh water velocity, U, is obtained by dividing fresh water discharge, Q, by the river cross-sectional, A. In the East River, fresh water flow is negligible and U is equal to zero. Fresh water flow into the Hudson is measured at Green Island, 152 miles above the Battery. Because fresh water flow in tidal waters cannot be measured, the Green Island gage is used to establish lower River discharges. The United States Geological Survey (USGS) has investigated the relation of lower River tributary flows to Green Island gage data; USGS indicated the most probable value of the ratio between yearly average lower River flows and yearly average Green Island gage readings is 1.22.

All values of lower River flow employed in this report were established using this ratio.

The pattern of the long term monthly flows, shown in Figure 17, is indicative of the general variation of River discharge. During the months of March through May, the flow averaged 29,000 cfs or almost 3.5 times the average discharge during the months from June through October. This is equivalent to the statement that the volume of fresh water discharged during the spring months is in excess of twice the volume discharged during the subsequent five month period.

Figure 1 and Equation 4 indicate that as fresh water velocity decreases, given a fixed value of the longitudinal dispersion coefficient, the dispersion effect increases. Therefore, contaminant concentration values upstream of a source of waste in the Hudson River can be expected to increase as flow decreases. Furthermore, due to increased salinity intrusion during periods of low fresh water flow, the longitudinal dispersion coefficient,


FLOW IN LOWER HUDSON RIVER
1918-1964 MONTHLY AVERAGE FLOW
1964 MONTHLY AVERAGE FLOW
FLOW IN LOWER HUDSON IN THOUSANDS CFS
40
38
36
34
32
30
28
26
24
22
20
18
16
14
12
10
8
6
4
2
0
JAN
FEB
MAR
APR
MAY
JUN
JULY
AUG
SEPT
OCT
NOV
DEC
QUIRK, LAWLER & MATUSKY

which is strongly dependent on salinity-induced circulation, increases in the Hudson River as fresh water flow decreases. For these reasons, analysis of the effect of pollutants on the River requires that drought flows be selected in assigning values of U.

Figure 18 shows a statistical analysis of Hudson River drought flows for the years 1918 through 1964. For drought durations of one week (seven consecutive days), and one month, a plot of flow versus the per cent of the time such flow can be expected to occur is given. For example, Figure 18 indicates, for a duration of one week, a flow of 2,630 cfs can be expected to occur five per cent of the time or once in 20 years.

In the following section, computer generated BOD and DO profiles are verified by comparison to observed concentrations. The year 1964 was chosen for this comparison because it represented a severe drought condition, and because extensive salinity data, collected during that period, yielded the most reliable estimate of dispersion coefficient. Figure 17 shows the lower River flow during the six month drought lasting from June through November, 1964 was 2,500 mgd or 3,900 cfs. Subsequent minor revision of the data presented in Figure 17 showed the average flow during this period was 4,100 cfs. This latter value has been used for the Hudson River flow throughout this study. Figure 18 shows the flow of 4,100 cfs represents the five year, one month drought; i.e., the flows of 4,100 cfs lasting for one month are expected to occur no more than once in five years.

The Arthur Kill is a tidal estuary and fresh water flow measurements are unavailable. Salinity measurements offer a means of evaluating the fresh water flow effect. The longitudinal salinity gradient in an estuary is proportional to fresh water flow. In the Arthur Kill, salinity concentration varies from 80% of sea water at Raritan Bay to 74% of sea water at Newark Bay or a 6% decrease over 13 miles. This flat gradient suggests the fresh water flow opposing salinity intrusion is relatively small.

Upland runoff to Newark Bay is largely flow from both the Hackensack and Passaic Rivers. Total flow into the Passaic River estuary is approximately equal to the sum of flows at gaging stations at Little Falls on the Passaic and at Lodi on the Saddle River. Average fresh water flow from August through November, 1964 was 95 cfs.

Hackensack River flow is measured at New Milford, below the Oradell Reservoir. The River is tidal below this point. Due to regulation and diversion, summer flows past the New Milford gage are negligible. For example, from June through September, 1964, fresh water flow at this gage averaged 0.75 cfs. An additional 20 cfs is estimated to be the drought runoff in the drainage area below this gage.

It is not known how these flows are distributed between the Arthur Kill and the Kill Van Kull. Since the Arthur Kill flow has been selected as zero in this study, and since the Hudson River flows are quite large by comparison to both Passaic and Hackensack River flows, these latter flows have been neglected in model computations in this study.

Fresh water flow into the Raritan is measured at Bound Brook, three miles above Fieldville Dam, the upper end of the tidal Raritan. The average flow at the gaging station from June through September, 1964 was 200 cfs. A lower average flow of 138 cfs was observed in June and July, 1966 and has been employed in some calculations on the Raritan in this study.

River Geometry - Top Width, Depth and Cross-Sectional Area

The area of any section was determined by constructing the section from sounding data and planimetering the boundary profile. Top width and soundings for various cross-sections of each River were obtained from U. S. Coast and Geodetic Survey maps.

Figure 19 shows the variation of cross-section area in the Hudson River with distance above the Battery. Variation is erratic and as such is not amenable to simple mathematical description; i.e., as an elementary function of distance. The average area of the River between the Battery and the City Line, mile point 16, is approximately 135,000 square feet or 0.00484 square miles.

In the East River, area is constant over the lower eight miles, but, above the junction with the Harlem River, increases steadily as it moves into Long Island Sound. For the constant parameter model, an average area for the upper portion was determined and was then averaged with the area in the lower portion. A value of 0.0047 square miles or 130,000 square feet results and has been used in this study for the cross-sectional area of the East River.

The volume of Upper New York Bay, bounded on the south by the Narrows, on the west by Kill Van Kull, on the north by the Hudson and on the east by the East River, was computed in the manner described above and found to be 12.7 billion cubic feet or 0.086 cubic miles.

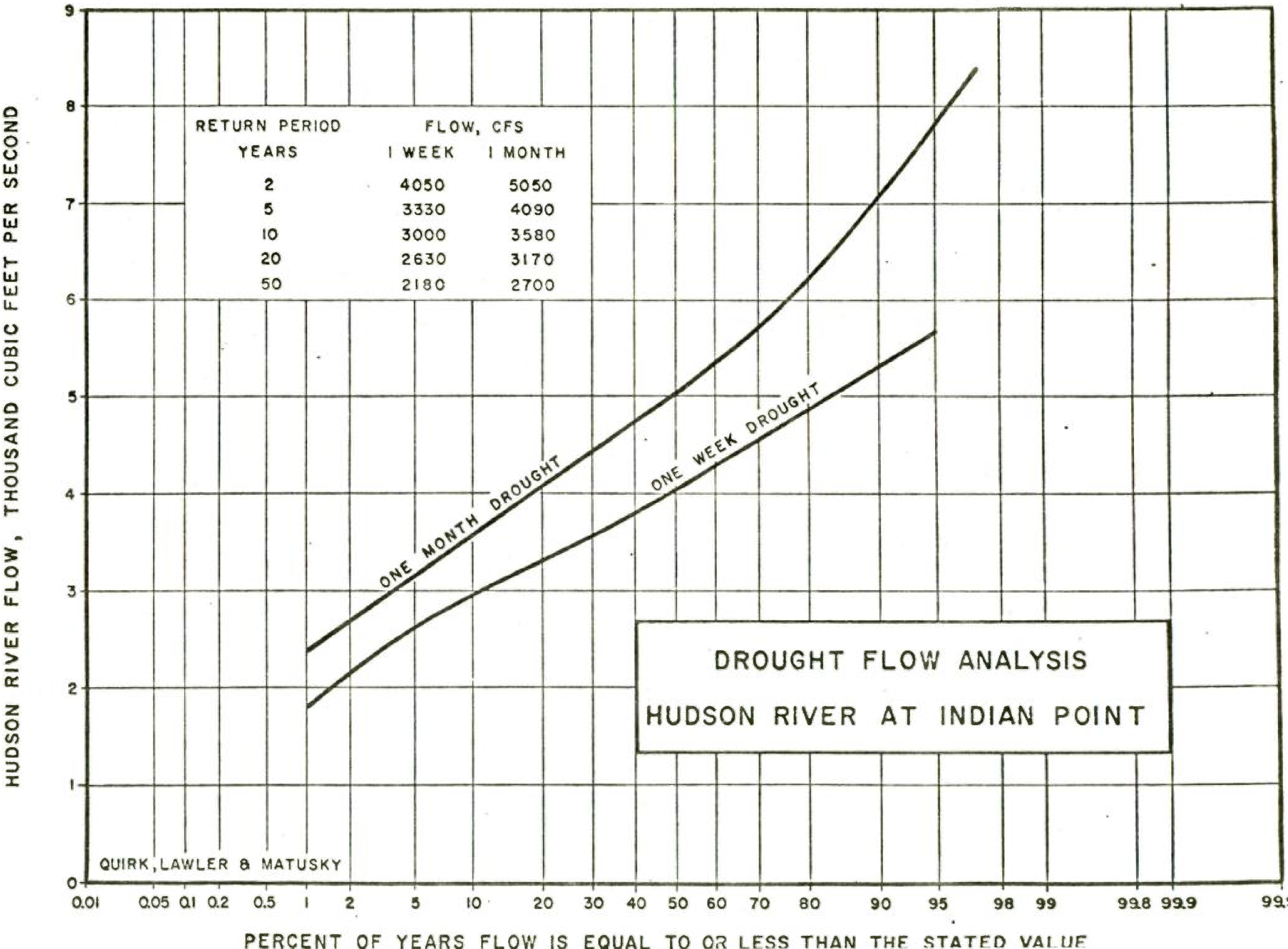
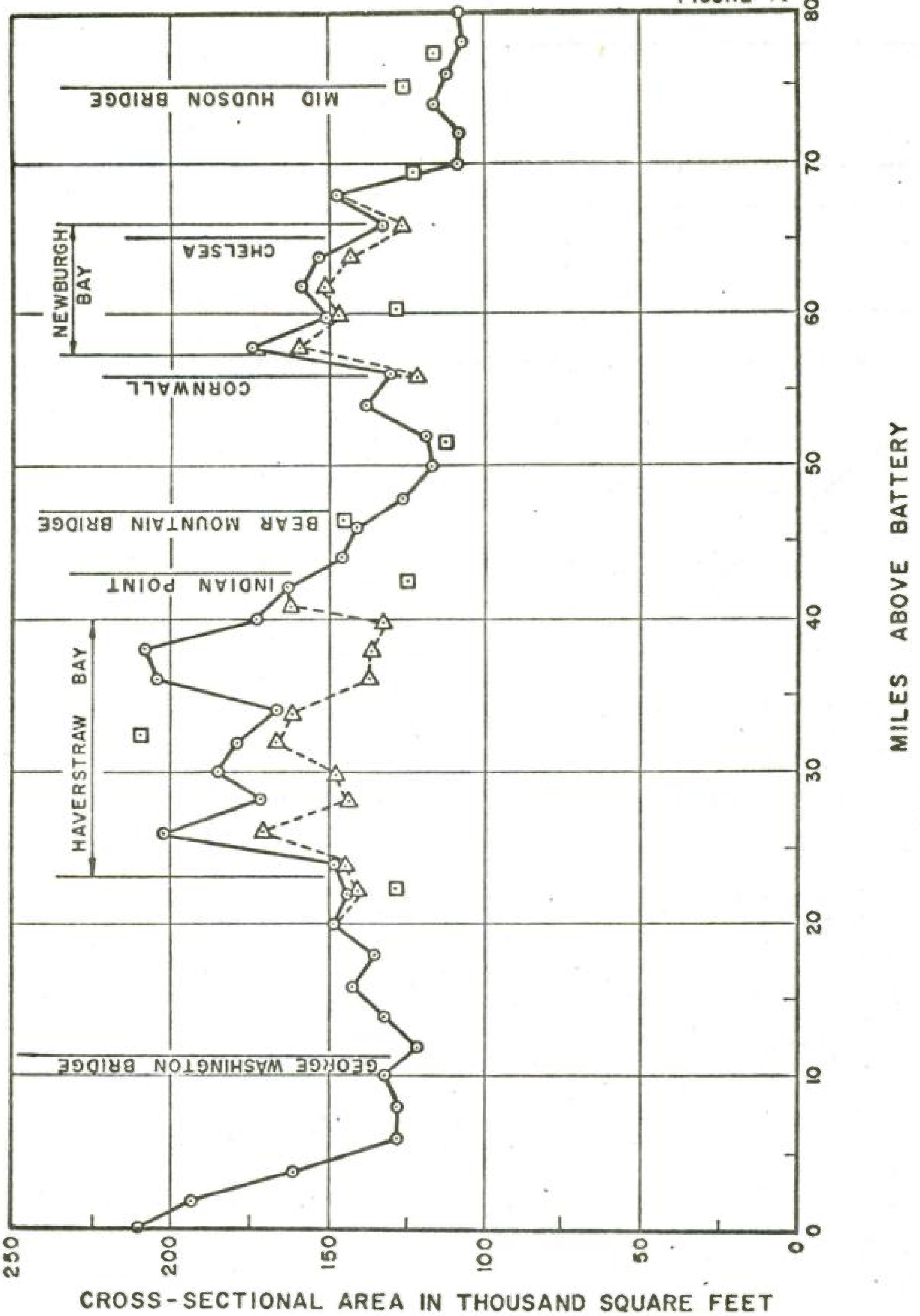

RETURN PERIOD YEARS
FLOW, CFS
1 WEEK
1 MONTH
2 4050 5050
5 3330 4090
10 3000 3580
20 2630 3170
50 2180 2700
ONE MONTH DROUGHT
ONE WEEK DROUGHT
DROUGHT FLOW ANALYSIS
HUDSON RIVER AT INDIAN POINT
QUIRK, LAWLER & MATUSKY
HUDSON RIVER FLOW, THOUSAND CUBIC FEET PER SECOND
0
1
2
3
4
5
6
7
8
9
0.01 0.05 0.1 0.2 0.5 1 2 5 10 20 30 40 50 60 70 80 90 95 98 99 99.8 99.9 99.99
PERCENT OF YEARS FLOW IS EQUAL TO OR LESS THAN THE STATED VALUE

FIGURE 18

FIGURE 19



⊙ TOTAL CROSS-SECTIONAL AREA, USC & GS SOUNDINGS

△ CHANNEL AREA, USC & GS SOUNDINGS

20'

⊡ TOTAL AREA, QL & M FATHOMETER SURVEY, 1964

HUDSON RIVER
CROSS - SECTIONAL AREA
MEAN LOW WATER

In the Arthur Kill, cross-sectional area does not vary significantly with distance. The average area of 35,000 square feet was used in this study.

Figure 20 shows the variation of Raritan River cross-sectional area with distance above Raritan Bay. An average area of 4,200 square feet was used for the lower Raritan River in this study. Although the constant area assumption is a gross approximation, its only use in this study is to bound the Arthur Kill model. Development of a transport model for the Raritan, using a variable area, would follow the procedures given in Section III.

Soundings are not available above Fieldville Dam. Upper Raritan River cross-sectional area was estimated from the area below the Dam and is 600 square feet.

The volume of Newark Bay, bounded on the south by the Arthur Kill, on the southeast by the Kill Van Kull, and on the north by the Passaic and Hackensack Rivers, was computed to be 2.6 billion cubic feet or 0.017 cubic miles.

For model use, the volume of Raritan Bay was defined between the junction of the Arthur Kill and Raritan River and a north-south boundary line drawn one mile east of Victory Bridge. The cross-sectional area within this reach is fairly constant. Beyond the one mile point, the area expands rapidly; for model purposes, this domain is considered to be the ocean. On this basis, the volume of Raritan Bay was found to be 0.027 cubic miles.

Longitudinal Dispersion Coefficient

The value of the longitudinal dispersion coefficient, E, at any point within a salt-intruded reach, can be conveniently obtained by analysis of salinity profiles. An example of one procedure for obtaining E from salinity profiles has been given in Section III. Procedures used to obtain E from salinity profiles in the subject waterways are given below.

The limiting form of Equation 2 for the case of a conservative substance such as salt, and non-constant values of Q, A and E, is:

$$\frac{1}{A}\frac{\partial}{\partial x}\left[EA\frac{\partial s}{\partial x} - Qs\right] = \frac{\partial s}{\partial t} \qquad (18)$$

If the variation of salinity with x and t is known, the derivatives $\frac{\partial s}{\partial x}$ and $\frac{\partial s}{\partial t}$ may be obtained graphically or numerically. Equation 18

RARITAN RIVER CROSS SECTIONS
VICTORY BRIDGE TO ALBANY ST. BRIDGE

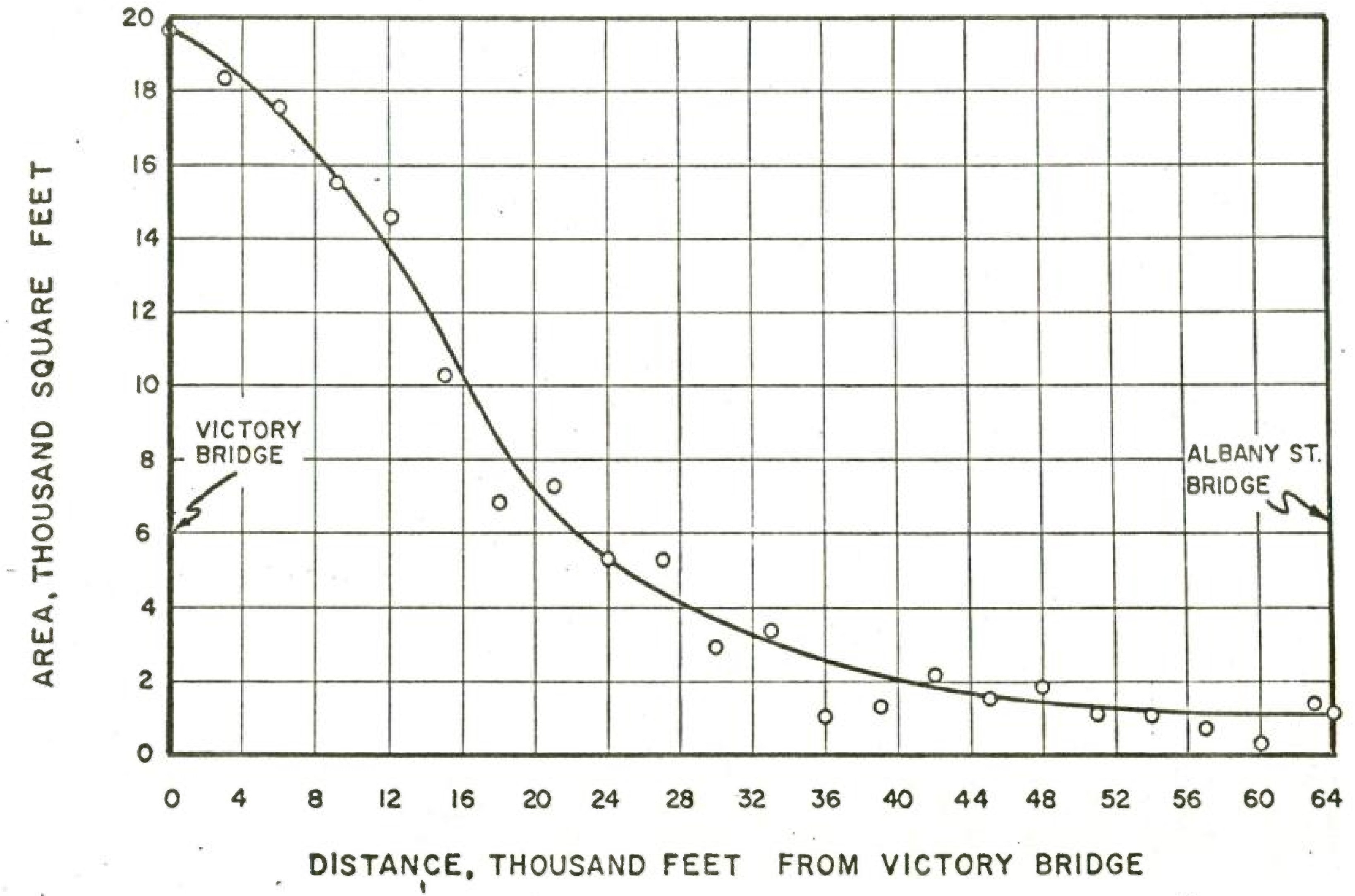


FIGURE 20

can then be used to compute the value of E at any point within the saline reach of the river.

This procedure requires that a number of profiles be available so that the time derivative, $\frac{\partial s}{\partial t}$, can be computed and also requires that the value of Q, now a time and distance dependent function, controlling the intrusion, be known. This latter requirement poses some difficulty in evaluating Hudson River dispersion. Fresh water flow can only be measured at Green Island, above the tidal region, and the attenuating effect of tidal mixing on time variable flows is not known.

These difficulties have been avoided by recognizing that drought flows in the Hudson remain relatively constant for extended periods of time; Q, and therefore U, are known and the steady Q gives rise to steady salinity profiles during these periods. Under these conditions, the net flux of salt in the River must be zero since there is no sink or source of salt within the estuary. Equation 18 then reduces to:

$$E \frac{ds}{dx} - \frac{Qs}{A} = 0 \qquad (19)$$

Rearrangement of Equation 19 yields a solution for the dispersion coefficient:

$$E = \frac{Q}{A} \left[2.303 \frac{d \log s}{dx} \right]^{-1} \qquad (20)$$

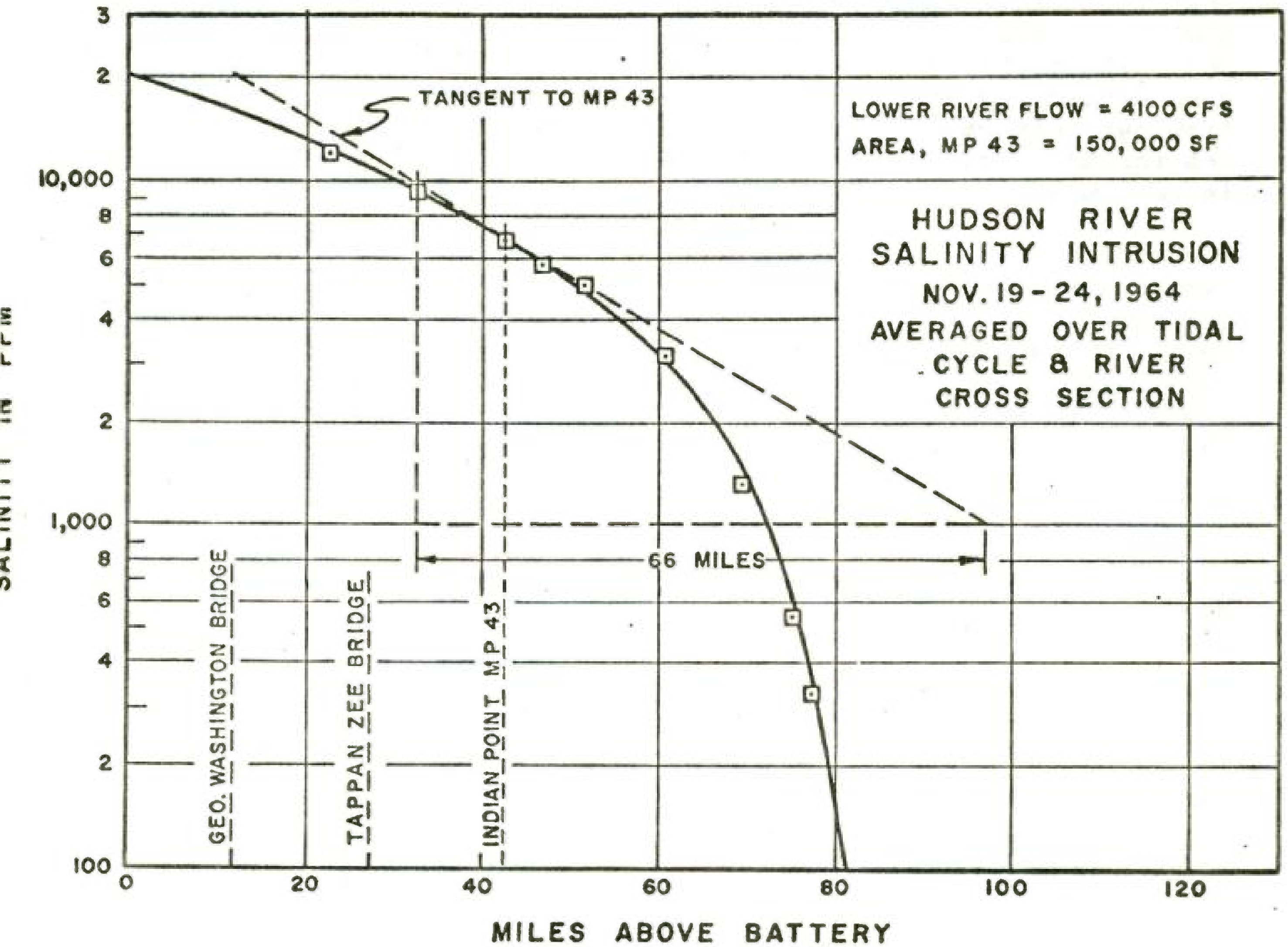
Numerical values of $\frac{d \log s}{dx}$ may be obtained by graphical differentiation of a semi-logarithmic plot of salinity versus distance. U(x) is equal to the flow associated with that profile, divided by the area, A(x), at the point in question.

Figure 21 shows the determination of the dispersion coefficient at Indian Point for the 1964 drought. Salinities are tidal-smoothed, area-averaged values and were computed from data shown on Figures 7 and 9 and similar measurements at other Hudson River stations.

Typical steady state salinity profiles for a number of Hudson River flows are shown in Figure 22. Values of E, computed as described above, are shown in Figure 23 for these and several other drought profiles.

FIGURE 21

CALCULATION OF DISPERSION COEFFICIENT
BY USE OF
STEADY STATE SALT FLUX EQUATION



$$E \frac{dC}{dx} - \frac{Q}{A} C = 0$$

$$E = \frac{Q}{A} \frac{C}{dC/dx} = \frac{Q}{A} \left[2.303 \frac{d \log C}{dx} \right]^{-1}$$

$$= \frac{4100}{150{,}000} \left[\frac{2.3(\log 10{,}000 - \log 1{,}000)}{5280 \times 66} \right]^{-1}$$

$$= 4140 \text{ SQ. FT.}/_{\text{SEC}} \quad \text{OR} \quad 12.8 \; \frac{\text{SQ. MILES}}{\text{DAY}}$$

FIGURE 22

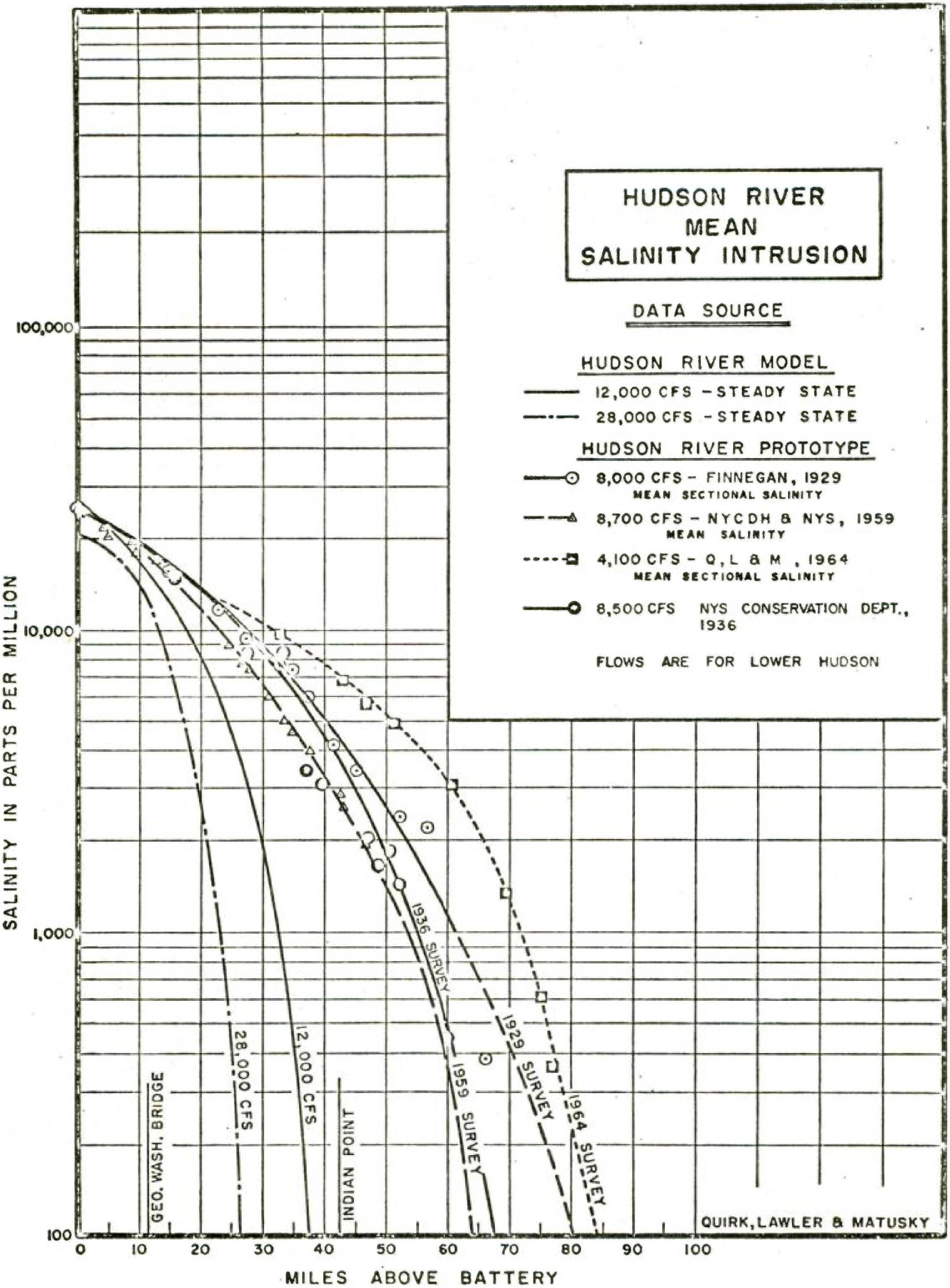

HUDSON RIVER
MEAN
SALINITY INTRUSION
DATA SOURCE
HUDSON RIVER MODEL
12,000 CFS - STEADY STATE
28,000 CFS - STEADY STATE
HUDSON RIVER PROTOTYPE
8,000 CFS - FINNEGAN, 1929
MEAN SECTIONAL SALINITY
8,700 CFS - NYCDH & NYS, 1959
MEAN SALINITY
4,100 CFS - Q, L & M , 1964
MEAN SECTIONAL SALINITY
8,500 CFS NYS CONSERVATION DEPT., 1936
FLOWS ARE FOR LOWER HUDSON
SALINITY IN PARTS PER MILLION
100,000
10,000
1,000
100
MILES ABOVE BATTERY
0
10
20
30
40
50
60
70
80
90
100
GEO. WASH. BRIDGE
INDIAN POINT
28,000 CFS
12,000 CFS
1936 SURVEY
1959 SURVEY
1929 SURVEY
1964 SURVEY
QUIRK, LAWLER & MATUSKY

FIGURE 23

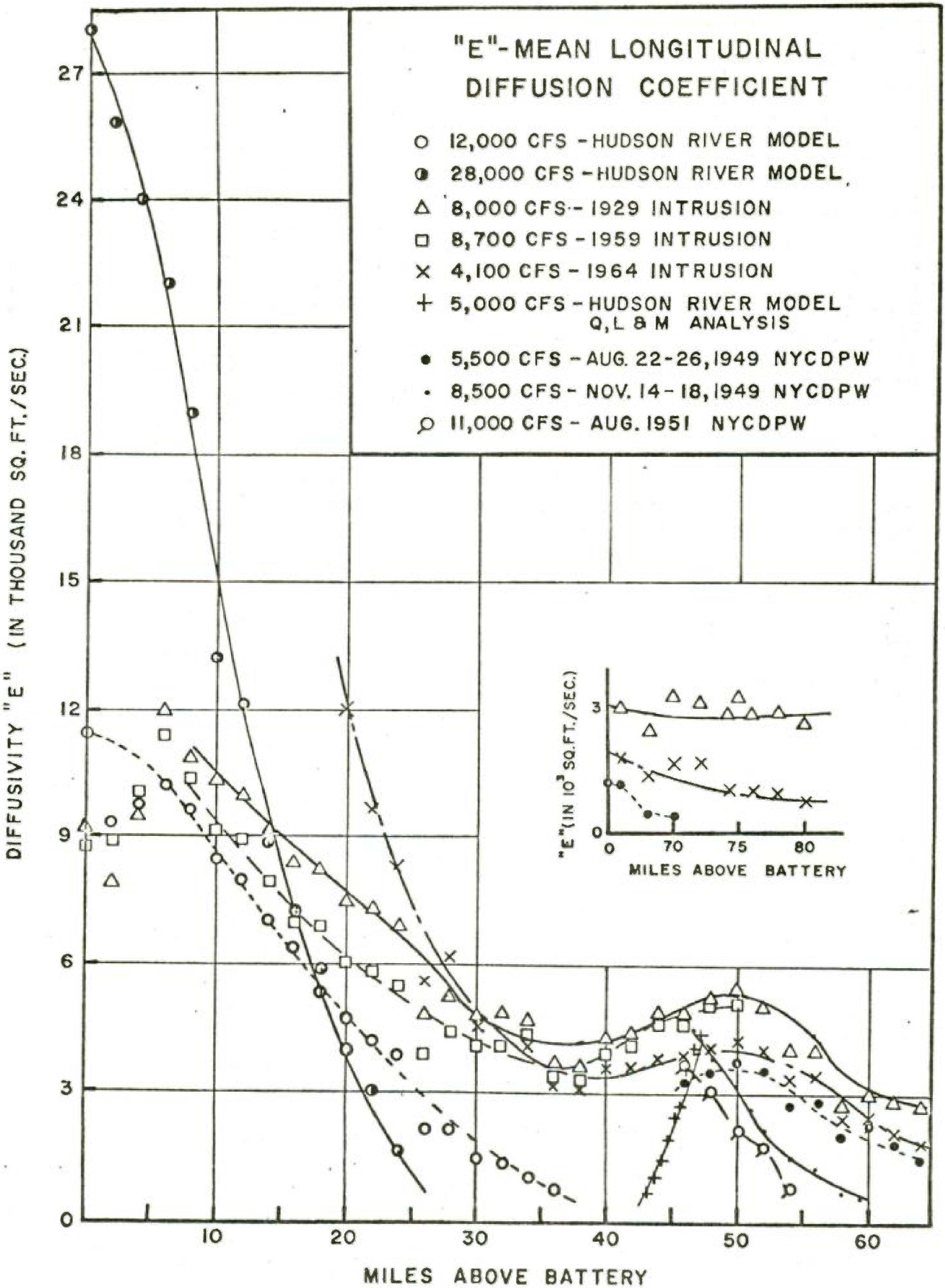

"E"-MEAN LONGITUDINAL DIFFUSION COEFFICIENT
12,000 CFS - HUDSON RIVER MODEL
28,000 CFS - HUDSON RIVER MODEL
8,000 CFS - 1929 INTRUSION
8,700 CFS - 1959 INTRUSION
4,100 CFS - 1964 INTRUSION
5,000 CFS - HUDSON RIVER MODEL Q, L & M ANALYSIS
5,500 CFS - AUG. 22-26, 1949 NYCDPW
8,500 CFS - NOV. 14-18, 1949 NYCDPW
11,000 CFS - AUG. 1951 NYCDPW
DIFFUSIVITY "E" (IN THOUSAND SQ. FT./SEC.)
MILES ABOVE BATTERY
"E" (IN 10³ SQ.FT./SEC.)
MILES ABOVE BATTERY
0
3
6
9
12
15
18
21
24
27
10
20
30
40
50
60
70
75
80

Figure 23 indicates the value of the dispersion coefficient in the 16 mile reach above the Battery varies between 7,000 and 12,000 square feet per second, depending on flow and location. These data are concentrated about the arithmetic average of 9,500 square feet per second or 30 square miles per day.

A second method of estimating the average dispersion coefficient over a finite reach of estuary applies the mean value theorem for derivatives to Equation 21. This yields:

$$\left[\frac{E}{U}\right]_{avg.} = \left[2.303 \frac{\Delta \log s}{\Delta x}\right]^{-1} \quad \text{.......................(21)}$$

Assume that the product of the averages closely approximates the average of the product.

$$E_{avg.} \doteq U_{avg.} \left[2.303 \frac{\Delta \log s}{\Delta x}\right]^{-1} \quad \text{...................(22)}$$

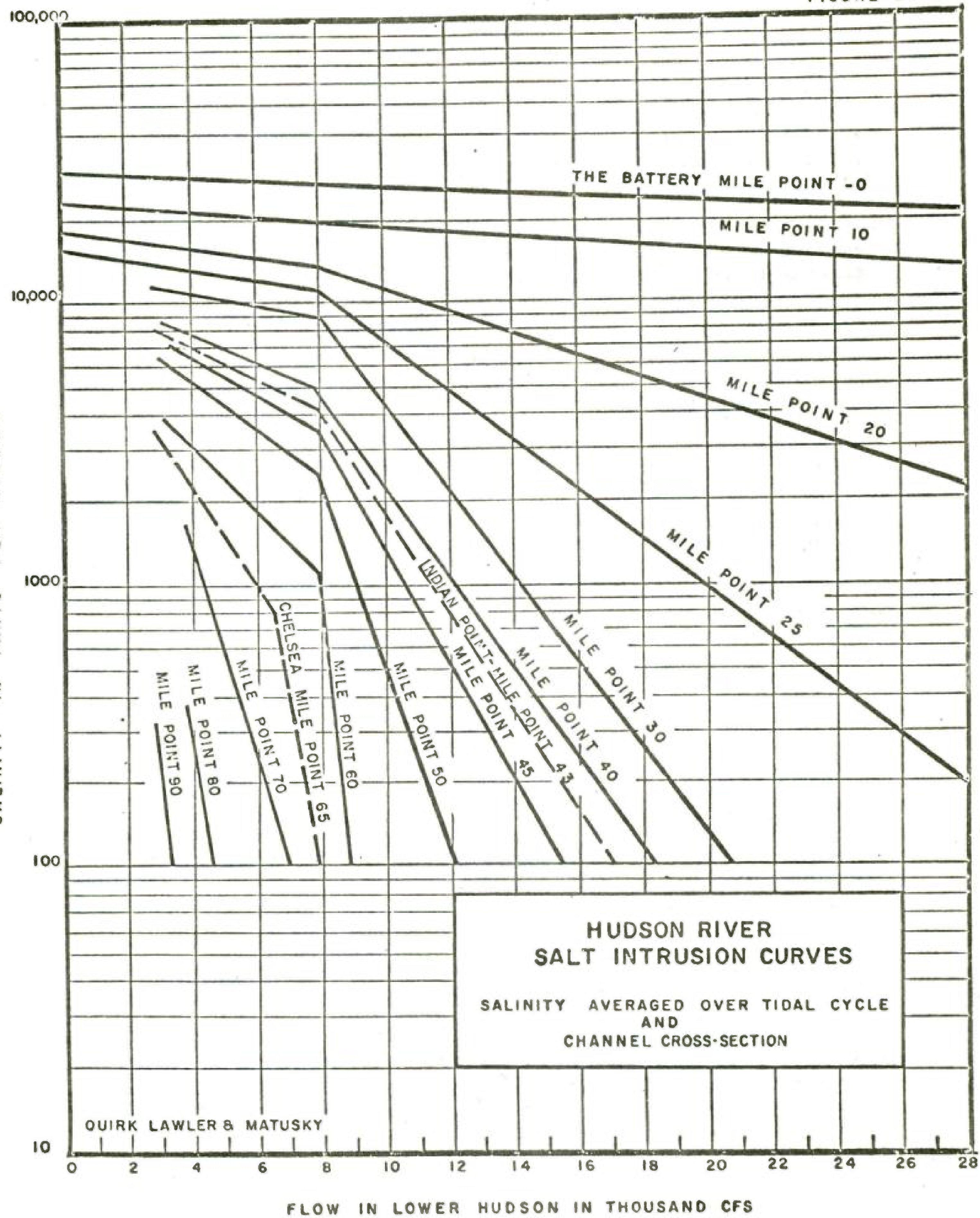
A correlation of all available Hudson River salinity and flow data is shown on Figure 24. Computation of average E between the Battery and mile point 20 for a flow of 4,100 cfs, by application of Equation 22 to Figure 24, yields a value of 23 square miles per day.

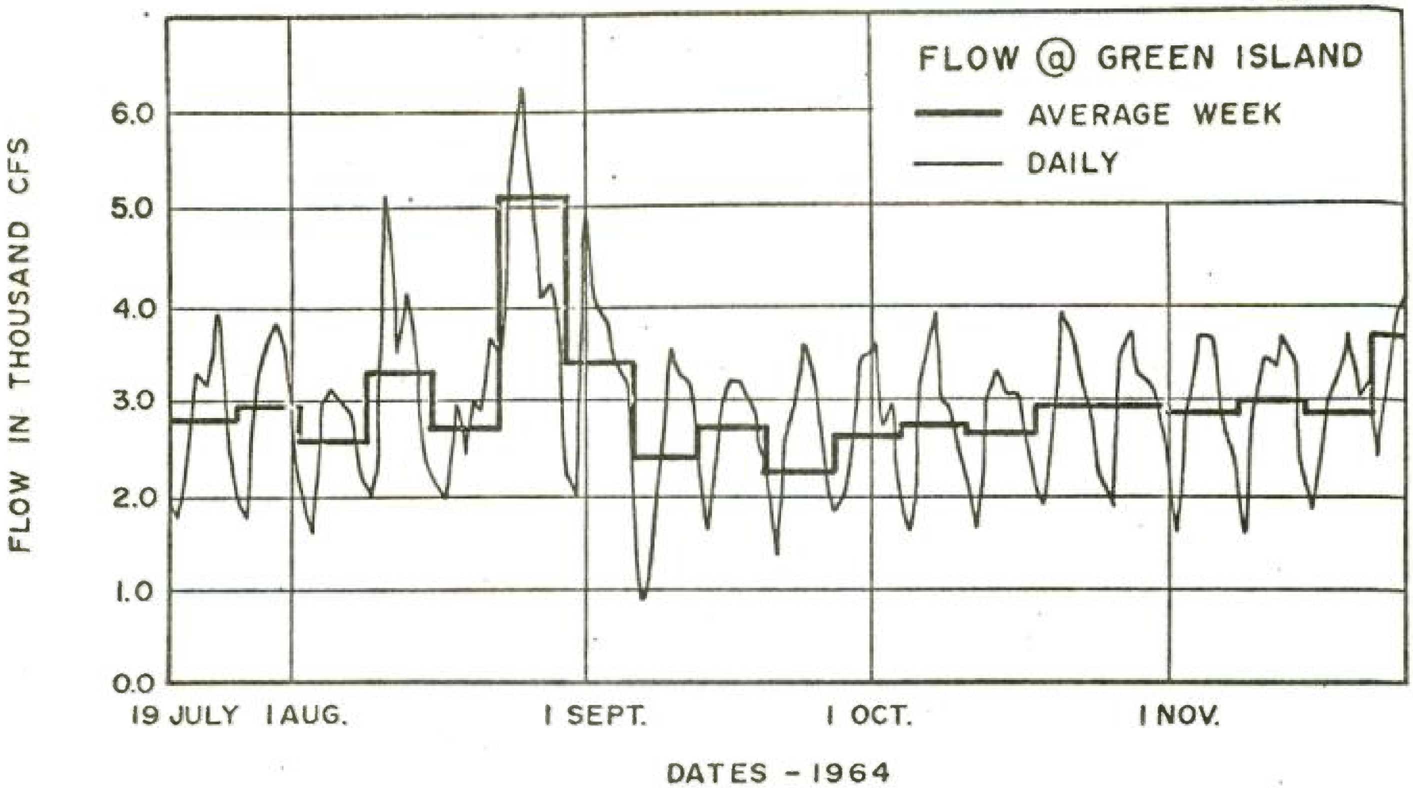
Figure 25 shows the salinity profile in the Hudson River towards the end of the 1964 drought. Application of Equation 22 to the extrapolated region of this profile between the Battery and mile point 20 yields an average E of 24 square miles per day.

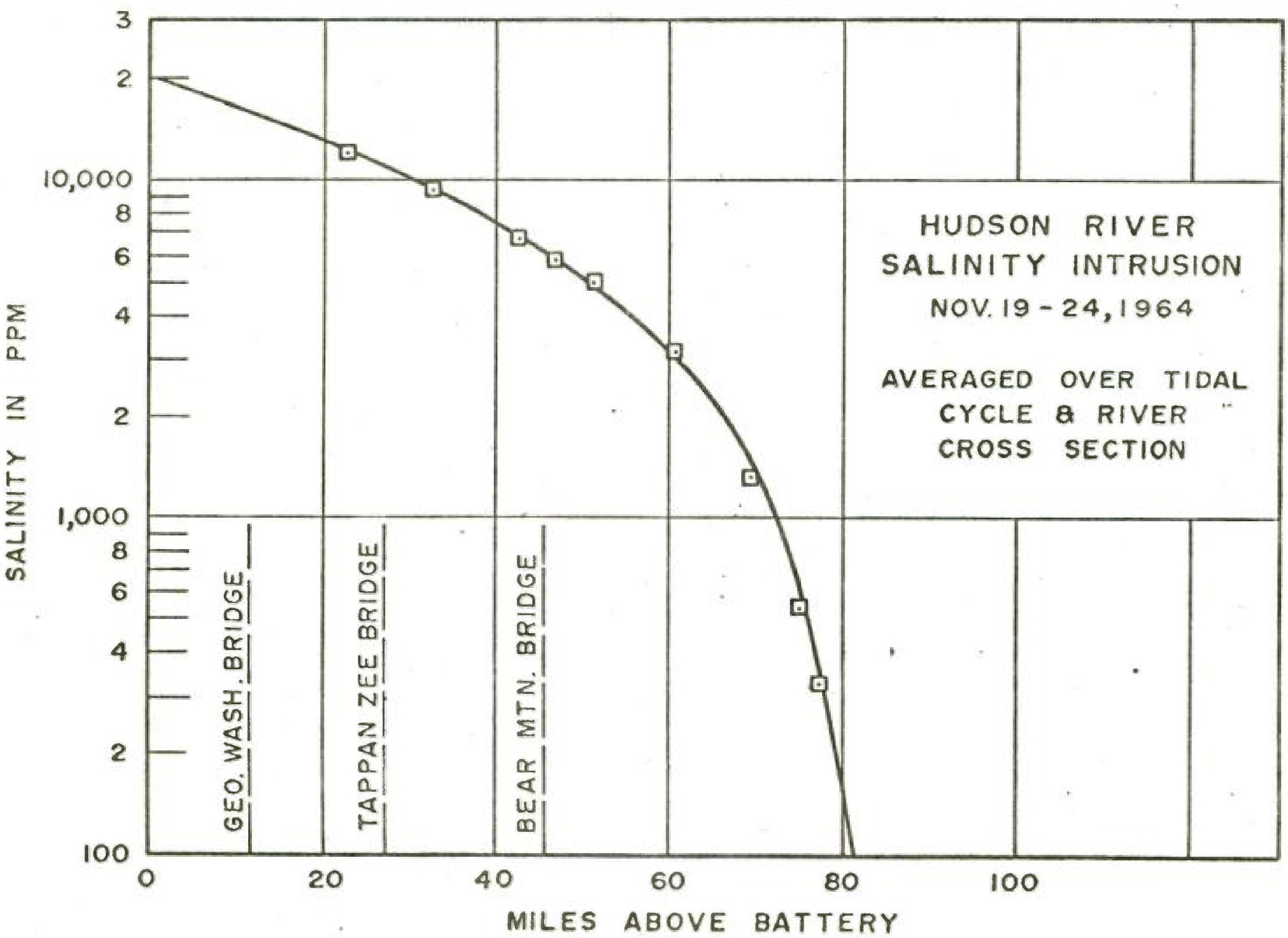
These lower estimates are not considered to be completely representative of the true average longitudinal mixing in the River below mile point 16. The profile in Figure 25 has been extrapolated into the reach in question to yield the value of 23 square miles per day. By comparison, direct application of Equation 22 to the measured profile above mile point 20 yielded, as shown in Figure 23, a point value of 12,000 sf/sec or 37 square miles/day at mile point 20.

E values below this point would be expected to be higher. For these reasons, the value of 30 square miles/day is considered to be a better estimate and, accordingly, has been selected for use in this study.

Sensitivity analyses of the dissolved oxygen model for the lower Hudson complex show that, for the relatively intense mixing levels of 20 to 40 square miles/day, model DO response is almost insensitive


100,000
10,000
1000
100
10
SALINITY IN PARTS PER MILLION
THE BATTERY MILE POINT -0
MILE POINT 10
MILE POINT 20
MILE POINT 25
MILE POINT 30
MILE POINT 40
INDIAN POINT-MILE POINT 43
MILE POINT 45
MILE POINT 50
MILE POINT 60
CHELSEA MILE POINT 65
MILE POINT 70
MILE POINT 80
MILE POINT 90
HUDSON RIVER
SALT INTRUSION CURVES
SALINITY AVERAGED OVER TIDAL CYCLE
AND
CHANNEL CROSS-SECTION
QUIRK LAWLER & MATUSKY
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
FLOW IN LOWER HUDSON IN THOUSAND CFS


FLOW @ GREEN ISLAND
AVERAGE WEEK
DAILY
FLOW IN THOUSAND CFS
6.0
5.0
4.0
3.0
2.0
1.0
0.0
19 JULY
1 AUG.
1 SEPT.
1 OCT.
1 NOV.
DATES - 1964


HUDSON RIVER
SALINITY INTRUSION
NOV. 19 - 24, 1964
AVERAGED OVER TIDAL
CYCLE & RIVER
CROSS SECTION
SALINITY IN PPM
3
2
10,000
8
6
4
2
1,000
8
6
4
2
100
GEO. WASH. BRIDGE
TAPPAN ZEE BRIDGE
BEAR MTN. BRIDGE
0
20
40
60
80
100
MILES ABOVE BATTERY

to changes in E. For a ten square mile/day increase in E, the average DO over the first 16 miles of the lower Hudson decreased only 0.75% saturation.

Only rough estimates of the dispersion coefficients for the East River and the Arthur Kill can be made by application of Equation 19. For longitudinal salinity gradients to develop in an estuary, fresh water flow must be sufficiently large to counteract tidal mixing. Equation 19 shows this clearly; for a given E, as U decreases, ds/dx must also decrease to maintain the equality. Fresh water flow in these waterways is relatively small and also difficult to estimate. (For model purposes, fresh water flows in both are set equal to zero.) By comparison, ocean salt enters each end of both of these waterways. This double-ended source of salt creates a flat gradient; salinity variation is slight and calculation of ds/dx is not precise.

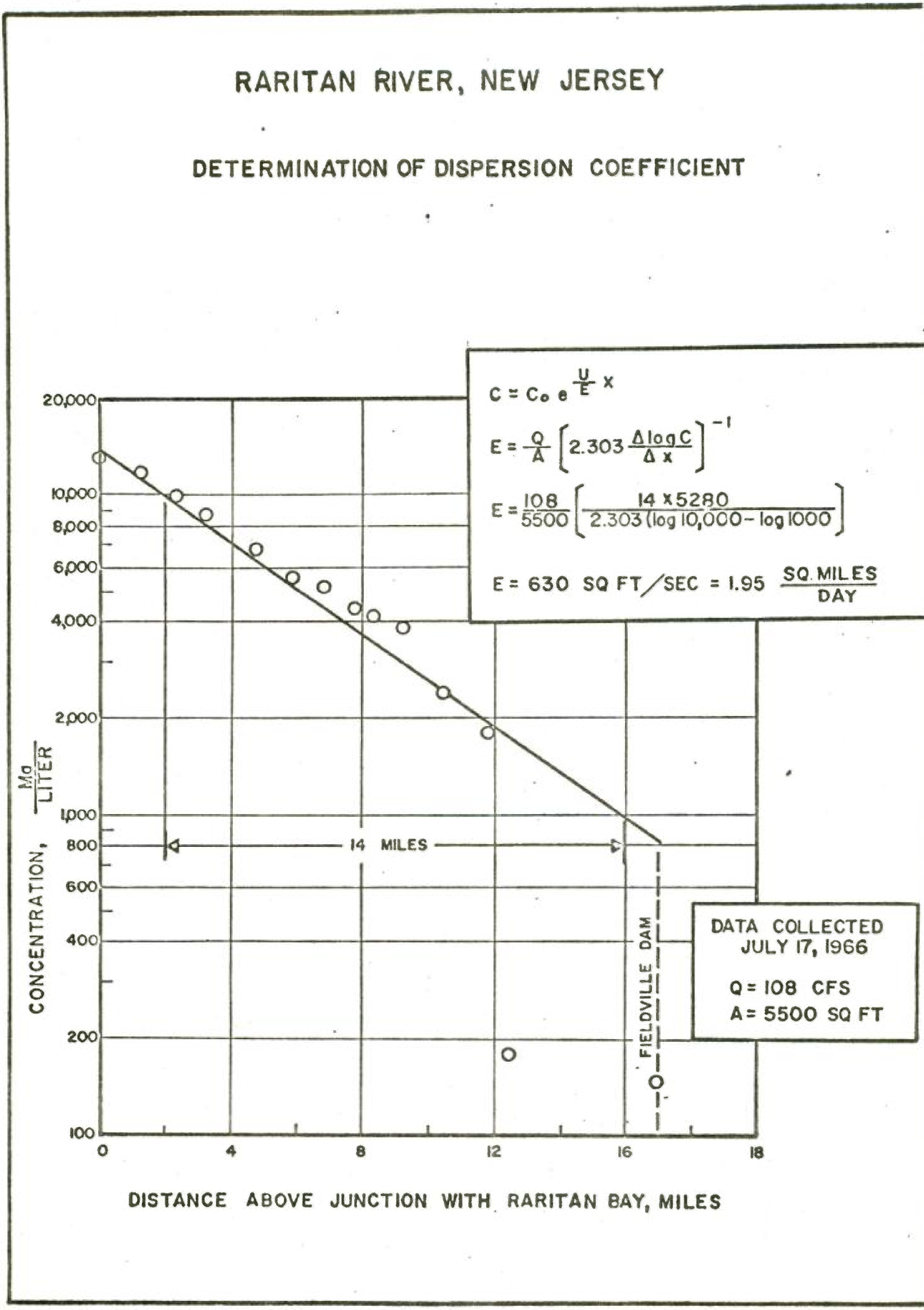
The dispersion coefficient for these waterways can also be estimated via tidal exchange data; however, precise information on this parameter is also lacking. In this study, values of ten and eight square miles/day were assigned to the Arthur Kill and East River, respectively. Although these values may be somewhat in error, DO response is relatively insensitive to changes in E and will not be proportionately affected.

A hydraulic model study of the dispersion of dye in the New York-New Jersey estuarine complex has been made by the Corps of Engineers (6). Dye was discharged continuously from several points within the complex and measurement of dye concentration was made throughout the estuary. These results have been subjected to mathematical analysis in an attempt to extract dispersion coefficients. Results showed higher dispersion coefficients for the Arthur Kill and East River than are known to prevail. Details of this analysis are not given in this report but are available at this office.

Salinity profiles for the Raritan River were constructed from salinity data collected during 1966 New Jersey State Department of Health Raritan River surveys. Evaluation of these salinity profiles resulted in an average longitudinal dispersion coefficient in the lower Raritan River of 2.33 square miles/day. Figure 26 shows a typical Raritan River salinity profile and an accompanying calculation of E.

<u>First Order Reaction Velocity Constant for BOD Decay, K</u>

Experimental determination of the reaction velocity constant, K,


RARITAN RIVER, NEW JERSEY
DETERMINATION OF DISPERSION COEFFICIENT
$C = C_0 e^{\frac{U}{E} x}$
$E = \frac{Q}{A}\left[2.303 \frac{\Delta \log C}{\Delta x}\right]^{-1}$
$E = \frac{108}{5500}\left[\frac{14 \times 5280}{2.303(\log 10{,}000 - \log 1000)}\right]$
$E = 630$ SQ FT/SEC $= 1.95 \frac{\text{SQ MILES}}{\text{DAY}}$
20,000
10,000
8,000
6,000
4,000
2,000
1,000
800
600
400
200
100
CONCENTRATION, $\frac{Mg}{LITER}$
14 MILES
FIELDVILLE DAM
DATA COLLECTED
JULY 17, 1966
Q = 108 CFS
A = 5500 SQ FT
0
4
8
12
16
18
DISTANCE ABOVE JUNCTION WITH RARITAN BAY, MILES

has not been made of the waters in question. Current field investigations by this office for the New York State Department of Health include measurement of this parameter in the Hudson River.

Values of K range between 0.2 and 0.7 day^{-1} (or 0.1 to 0.3 day^{-1} on a common log basis) (7); the vast majority of rivers have values closer to the lower limit. In studies (8) in a grossly polluted fresh water stream, K varied between 0.21 and .53 day^{-1} and appeared to be strongly dependent on concentration of organisms. In previous studies in New York Harbor (9), (10), a value of 0.23 day^{-1} at 68°F was selected for the East River and Upper New York Bay and adjusted to 0.25 day^{-1} to account for prevailing temperature. A value of 0.25 day^{-1} was used in a previous study of the overall New York-New Jersey Harbor Complex (1); good agreement between computed and predicted profiles confirmed the selection of this value.

A value of 0.25 day^{-1} appears to represent behavior in the subject waterways. With the exception of the Arthur Kill, the decay rate of 0.25 day^{-1} has been employed throughout this study. Since the Arthur Kill is grossly polluted, a value of 0.5 day^{-1}, representative of a greater level of biological activity, has been selected for the reaction velocity constant for this reach.

Reaeration Coefficient, K_2

The reaeration coefficient, K_2, the unit rate at which oxygen is transferred into water from the atmosphere, varies directly with river velocity and inversely with river depth. K_2 can be estimated by the following relationship:

$$K_2 = \frac{(D_L U)^{1/2}}{H^{3/2}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (23)$$

in which:

K_2 = mean tidal velocity, ft/sec
H = mean depth, ft.
D_L = molecular diffusivity of oxygen in water
U = mean tidal velocity, ft/hour

For sinusoidal variation, mean deflection is equal to $2/\pi$ times the amplitude. Mean tidal velocity at six stations in the harbor complex was estimated as $2/\pi$ times the arithmetic average of the maximum ebb and flood currents at each of these stations, as given in the Tidal Current Tables for 1964 (11). Mean depth was obtained by dividing cross-sectional areas within the reach in question by the corresponding top widths. D_L, the molecular diffusion coefficient of oxygen in water, is 8.1×10^{-5} ft^2/hour.

Equation 23 was programmed on a digital computer, and the reaeration coefficient for each cross-section determined; the arithmetic average of these values for any given reach was selected as the working value of the reaeration coefficient for that river. Table 1 lists the reaeration coefficients for each reach in the study.

A comparison of the reaeration coefficients, computed by this technique and employed in this study, to previously published values (9), (10) for the lower Hudson River, the East River, and Upper New York Bay is shown below.

Comparison of Computed K_a Values

	K_a, Reference (9)	K_a, This Study
Hudson	.09	.105
New York Bay	.20	.150
East River	.08 Lower .11 Upper	0.098 Lower and Upper

Dissolved Oyxgen Saturation Concentration, C_s

The dissolved oxygen saturation value depends on the water temperature and the chloride concentration. Water temperatures during the period, June through September, 1964 were obtained from the New York City Department of Public Works 1964 Harbor Survey Report (12). The average water temperature during this period was computed to be 70°F. Salinity in the lower Hudson during periods of low flow averages approximately 20,000 ppm; the corresponding chloride concentration is 11,000 ppm. For these values of temperatures and chlorides, C_s is 8.00 mg/liter (13). For investigation of other conditions of temperature and salinity, required in the Arthur Kill analysis, correct values of C_s were obtained from reference 13.

Tidal Exchange Coefficient, R

Development of boundary conditions, discussed in Sections V and VI, require use of the tidal exchange coefficient. This coefficient is a measure of the exchange of water between any two adjacent

TABLE 1

REAERATION COEFFICIENT, K_2
IN THE
NEW JERSEY-NEW YORK ESTUARINE COMPLEX

Reach	K_2 (Day^{-1})
Hudson River	0.105
Upper New York Bay	0.150
East River	0.098
Arthur Kill	0.167
Lower Raritan River	0.213
Upper Raritan River	1.62
Newark Bay	0.39
Raritan Bay	0.214

reaches of a waterway during a tidal cycle.

The tidal exchange coefficient is calculated by dividing the intertidal volume, ΔV, by the mean volume, V, of a segment of the estuary (14). The intertidal volume, ΔV, is defined as the difference in the volume of water between high and low tides for the section of estuary under consideration.

Tidal exchange measures the dilution available through exchange of polluted estuary water with cleaner sea water; multiplication of the exchange coefficient as defined above, by the difference in BOD between two adjacent segments, will yield the net transport of pollutants out of the estuary per cycle.

Section VI shows that exchange coefficients are required between Upper New York Bay and Lower New York Bay, and between Raritan Bay and the ocean. Estimation of these coefficients is given below.

Mean tidal range of the Battery is 4.4 feet and mean depth of Upper New York Bay is 47 feet; hence the computed exchange coefficient is 0.09 per tidal cycle or 0.18 per day. Mean tidal range at South Amboy is five feet and mean depth of Raritan Bay is 37 feet; hence the computed exchange coefficient is 0.13 per tidal cycle or 0.26 per day.

Ketchum (14) indicates this approach yields results which agree with the measured exchange for all but the seaward end of the estuary, at which location the calculated value will be exaggerated. For this reason, an exchange coefficient of 0.1 day^{-1} has been employed throughout this study.

Distribution of Waste Loads

Waste in the form of BOD enters the subject waterways from New Jersey, New York City and upper New York State. Table 2 lists loads presently discharged to the Arthur Kill and Raritan Bay. Table 3 lists loads presently discharged to the Hudson River and Upper New York Bay. East River loads are given in Table 4.

Data on New Jersey loads were developed by the Division of Pure Air and Water, New Jersey State Department of Health. All known outfalls were located; measured or estimated values of flow and waste strength were tabulated for both municipal and industrial loads.

TABLE 2

PRESENT BOD DISCHARGE, #/DAY, TO THE ARTHUR KILL

New Jersey Sources	Distance from the Raritan Bay (Miles)		Discharge #BOD/Day
Middlesex County	0.0	to	145,000
Perth Amboy	0.0	Raritan	16,400
South Amboy	0.0	Bay	707
Sewaren	3.3		2,500
Hess Oil	4.55		10,500
Carteret	6.64		2,430
FMC Corporation	7.1		250
Rahway Valley	7.75		23,600
American Cyanamid	8.05		1,600
Cities Service	8.8		6,300
Linden-Roselle	9.05		41,000
General Analine	9.25		9,500
Dupont	9.45		2,000
Humble Oil	9.8		80,000
Elizabeth Joint Meeting	11.0		101,000
Total			442,787
New York City Sources			
Staten Island	0.0		13,500
Untreated	5.5		13,500
	8.0		13,500
Procter and Gamble	12.75		20,000
			60,500

TABLE 3

PRESENT BOD DISCHARGE, #/DAY,
TO THE HUDSON RIVER

New York City Sources	Distance from the Battery (Miles)	Discharge #BOD/Day
Owls Head Treatment Plant and Staten Island Untreated	0.0	62,900
Manhattan Untreated	0.492	18,000
	1.818	34,500
	3.466	58,000
	4.905	51,000
	6.326	41,000
	7.538	31,500
	9.470	31,500
	11.136	19,100
	12.9	8,000
Riverdale - Marble Hill Untreated	14.0	5,450
Total		360,950
New Jersey Sources		
Passaic Valley Sewerage Commission	0.0	650,000
Jersey City East	0.1	37,600
Hoboken	2.8	28,000
West New York	5.7	9,000
North Bergen Township	6.4	860
Edgewater	7.5	2,767
Total		728,227
Upper New York State Sources		
Yonkers Municipal	17.0	70,500
Rockland Municipal and Industrial	27.0	10,300
Westchester Municipal and Industrial	39.0	67,000
Total		147,800

TABLE 4

PRESENT BOD DISCHARGE, #/DAY,
TO THE EAST RIVER

New York City Sources	Distance from the Battery (Miles)	Discharge #BOD/Day
Brooklyn and Manhattan Untreated	0.27	43,000
	1.212	26,000
	1.818	29,000
	2.368	74,000
	2.803	41,500
	3.371	31,500
Newtown Creek	4.109	110,000
Brooklyn Untreated	4.659	33,200
	5.208	18,600
	5.833	25,000
Wards Island Treatment Plant	8.6	58,000
Bowery Bay Treatment Plant	10.4	52,000
Hunts Point Treatment Plant	10.8	26,600
Tallmans Island Treatment Plant	13.1	16,200
City Island and Harts Island	14.0	1,000
Total		585,600

TABLE 5

PRESENT BOD DISCHARGE, #/DAY,
TO THE HUDSON RIVER,
EAST RIVER AND UPPER NEW YORK BAY

Source	Hudson River	East River	Upper Bay	Total
New York City Treated	--	153,800	55,000	208,800
New York City Untreated	298,050	431,800	7,900	737,750
New Jersey	78,227	--	650,000	728,227
Upstate New York	147,800	--	--	147,800
	524,077	585,600	712,900	1,822,577

The Middlesex County Sewerage Authority load was computed from data obtained in the Eighth Annual Report of the Middlesex County Sewerage Authority, 1965. Annual average flow and effluent BOD concentrations were reported for the years 1961 through 1965 on the basis of an average effluent BOD of 375 ppm and an average flow of 45.8 mgd. The five year average load was 145,000 #BOD/day.

The Passaic Valley Sewerage Commission load was computed from measurements made of the plant by the NJSDH on September 15, 1965. These data yield a composite effluent BOD of 350 ppm, an average flow of 221 mgd and a total load of 650,000 #BOD/day.

The existing New York City waste load consists of residual BOD contained in the effluent from the NYCDPW Pollution Control Projects and of raw BOD discharged from outfalls in the untreated sections of the City. Average waste load leaving each plant over the period June through September, 1961-1965, was computed using the Department of Public Works' Pollution Control Project Operating and Efficiency Summaries.

Waste loads from the untreated domestic and commercial areas were computed using the NYCDPW population estimates for the area, a flow equivalent of 200 gpcd and an average raw waste BOD concentration of 132 mg/liter. This concentration is the average influent BOD which can be expected in a residential-commercial area; selection of this value and detailed analysis of New York City loads is discussed in a prior report (1).

The BOD load in New York State municipal wastes discharged to the Hudson above the City Line was determined using a population equivalent factor of .17 #BOD/day/capita. The industrial loads were estimated with the assistance of personnel from the White Plains Regional Office of the New York State Department of Health.

V. Single Source, Single River System

This section develops the BOD and DO profiles for the case of a source of waste discharged to an estuary at a single point x. The estuary is assumed to be infinite in extent in both upstream and downstream directions so that transport is wholly with a single waterway. For illustrative purposes, the modified behavior of the profiles when the reach is bounded at one end is also given.

The purpose of this section is three-fold:

1. To present the rationale behind the assignment of boundary conditions. This is most readily done by consideration of the simplest case, the receiver of infinite extent.

2. To show the functional dependence of the solution, i.e., BOD and DO profiles, on the waste loading and on river system parameters.

3. To evaluate numerically the response of BOD and DO to typical individual waste loadings in the subject waterways. The solution for single source in the receiver of infinite extent is particularly valuable when applied at the point of waste discharge; i.e., the maximum BOD due to a given source always occurs at the point of discharge, and can be closely estimated, in any case in which the discharge does not occur close to an end of the waterway, by the solution for the receiver of infinite extent.

Steady State BOD Profile - Single Plane Source of Waste, Discharged Continuously to a Tidal River of Infinite Length

This situation is illustrated in Figure 5. The defining differential equation is the steady state form of Equation 4:

$$E\frac{d^2L}{dx^2} - U\frac{dL}{dx} - KL = 0 \qquad (24)$$

The discontinuity at the point of discharge occurs because the differential equation was developed by inventorying mass over any finite volume element within the region under consideration and then shrinking the dimensions of that element to zero. Since the location of the element is arbitrary, no source term is included in the inventory. At the point x=0 however, a source does exist so the solution cannot be expected to be continuous in all its derivatives across this point. Thus, the solution is broken up into two regions. Since integration of Equation 24 will yield two integration constants, each reach will require two boundary conditions. The required four boundary conditions are developed as follows:

1. The receiver of infinite extent implies that the contaminant can be expected to reach negligible concentrations before passing out of the estuary into the ocean. This is not due to any diluting effect of the ocean, but rather because the distance between the point of discharge and the ocean is sufficiently long to permit virtually complete disappearance of contaminant originating at x=0 by the time this contaminant reaches the ocean. This means that the downstream end of the estuary has no influence on contaminant distribution in the estuary. The estuary therefore is considered to be infinitely long and the first boundary condition is written:

$$L_{II}\Big|_{x=\infty} = 0 \quad \text{.............................. BC \#1}$$

2. In the upstream region, convection opposes dispersion so that the actual distance to the upstream end of the estuary, for the upstream boundary to have no influence on the contaminant concentration, is even less than that for the downstream end. The statements concerning BC #1, therefore, apply to the upstream boundary also, and the second boundary condition is written:

$$L_{I}\Big|_{x=-\infty} = 0 \quad \text{.............................. BC \#2}$$

3. Although Equation 25 does not define behavior across the plane of discharge, and discontinuity in some derivatives will occur at these points, the contaminant concentration itself is continuous, and therefore single-valued at all points. This fact gives rise to the third boundary condition:

$$L_{I}\Big|_{x=0} = L_{II}\Big|_{x=0} \quad \text{.............................. BC \#3}$$

4. To describe the behavior at the boundary between regions I and II, a material balance about the plane of discharge is constructed as shown on Figure 27. The steady state material balance is written:

$$\left[Q_I L_I - EA\frac{dL_I}{dx}\right]_{x=-\frac{\Delta x}{2}} + Q_W L_W - \left[Q_{II} L_{II} - EA\frac{dL_{II}}{dx}\right]_{x=\frac{\Delta x}{2}} = K\bar{L}A\Delta x \quad \text{......(25)}$$

Simplifying Equation 25 and taking the limit as $\Delta x \rightarrow 0$ yields:

$$Q_W\left[L_W - L_I\right]_{x=0} = EA\left[\frac{dL_I}{dx} - \frac{dL_{II}}{dx}\right]_{x=0} \quad \text{.............(26)}$$

Normally, the estuary fresh water flow, Q_I, is much greater than the waste flow, Q_W. For this case, $Q_I = Q_{II}$. Call $(Q_W \cdot L_W)$, W, the continuous load on the river, take the limit of Equation 25 and obtain for the fourth boundary condition:

$$W = EA\left[\frac{dL_I}{dx} - \frac{dL_{II}}{dx}\right]_{x=0} \quad \text{..................BC \#4}$$

The discontinuity in the first derivatives of the contaminant concentration at the point of discharge is shown clearly by the contaminant profile in Figure 1. dL_I/dx is always positive while dL_{II}/dx is negative. Thus, at x=0, BC #4 states that the waste load W is transported away from the plane of discharge, in the upstream direction with a rate equal to EA dL_I/dx and in the downstream direction with a rate equal to EA dL_{II}/dx.

To reduce the number of parameters carried through the various mathematical manipulations, Equation 24 and the four boundary conditions are written in dimensionless form. The procedure for selection of dimensionless variables used in this work is illustrated using the foregoing system.

CONTINUOUS CONTAMINANT DISCHARGE INTO A RECEIVER OF INFINITE EXTENT

MATERIAL BALANCE ABOUT THE POINT OF DISCHARGE

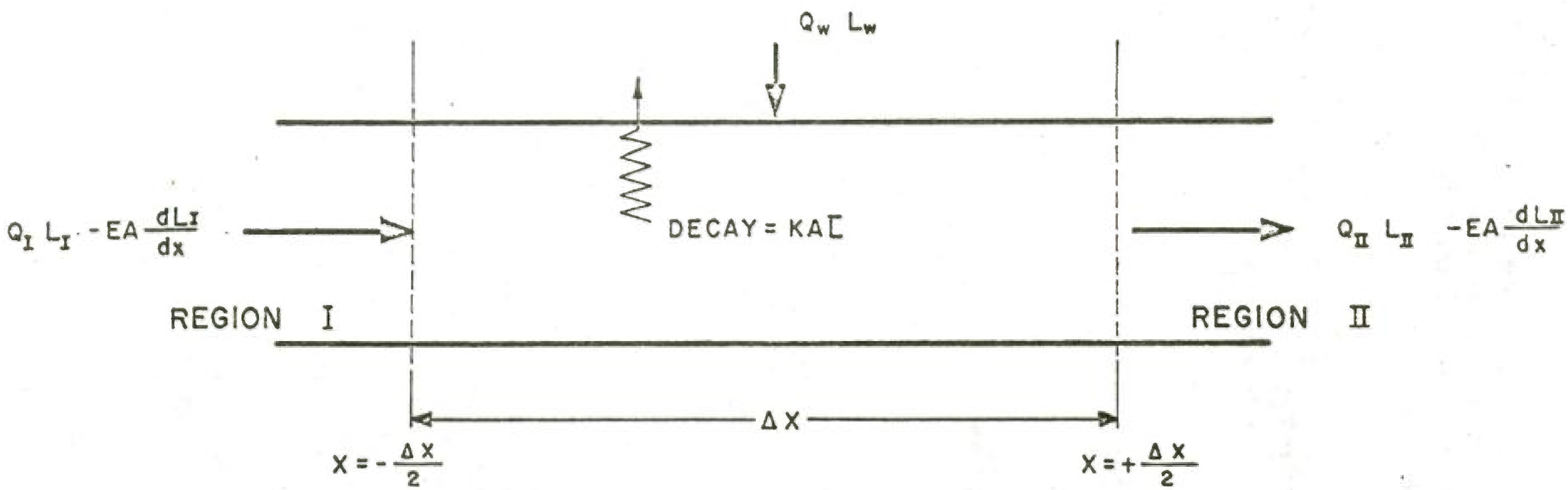


FIGURE 27

In Equation 24, replace x by $X\xi$ and L by $L_o \Gamma$. X and L_o are an as yet undefined length and concentration characteristic of the system, are fixed, and have the dimensions of distance and concentration, respectively. ξ and Γ are dimensionless distance and concentration variables. Equation 24 becomes:

$$\frac{d^2\Gamma}{d\xi^2} - \left(\frac{UX}{E}\right)\frac{d\Gamma}{d\xi} - \left(\frac{KX^2}{E}\right)\Gamma = 0$$

............(27)

Since X is as yet undefined, choose X so that either the coefficient of the second or third term in the above equation is unity. Arbitrarily, let KX^2/E equal unity, X becomes $\sqrt{E/K}$ and ξ is completely defined. A dimensionless group, $U/\sqrt{KE}$, appears in the dimensionless differential equation, and is written N. L_o is still undefined, the parameters W and A have not appeared as yet in the dimensional analysis, and the boundary conditions have as yet to be made dimensionless. The first three conditions can be put in dimensionless form immediately but do not aid in the selection of L_o. Replacing x and L in the fourth condition with $\sqrt{E/K}\xi$ and $L_o\Gamma$ yields:

$$\frac{W}{A\sqrt{KE}} = L_o\left[\frac{d\Gamma_I}{d\xi} - \frac{d\Gamma_{II}}{d\xi}\right]_{\xi=0}$$

Simplicity suggests that L_o be selected as $\frac{W}{A\sqrt{KE}}$. The mathematical description of the system is now completely defined in dimensionless terms, i.e., Equation 24 and the associated boundary conditions have dimensionless counterparts in terms of ξ, Γ and N and all of the original system variables and parameters have been used in defining these dimensionless quantities. The dimensionless system is summarized in Table 6.

The defining differential equation is an ordinary, linear, homogeneous, second order equation with constant coefficients. Its general solution* is:

$$\Gamma = C_1 e^{J_1\xi} + C_2 e^{J_2\xi}$$

......................(29)

*For a discussion of the general solution of the equation, see any elementary text on ordinary differential equations.

TABLE 6

DIMENSIONLESS STATEMENT FOR
SINGLE PLANE SOURCE OF BOD
IN AN INFINITE RECEIVER

ODE $$\frac{d^2\Gamma}{d\xi^2} - N\frac{d\Gamma}{d\xi} - \Gamma = 0 \quad \ldots\ldots(28)$$

BC #1 $$\Gamma_{II}\Big\}_{\xi = \infty} = 0$$

BC #2 $$\Gamma_{I}\Big\}_{\xi = -\infty} = 0$$

BC #3 $$\Gamma_{I}\Big\}_{\xi = 0} = \Gamma_{II}\Big\}_{\xi = 0}$$

BC #4 $$\left[\frac{d\Gamma_I}{d\xi} - \frac{d\Gamma_{II}}{d\xi}\right] = 1$$

$$\Gamma = \left(\frac{A\sqrt{KE}}{W}\right)L$$

$$\xi = \sqrt{K/E}\ x$$

$$N = \frac{U}{\sqrt{KE}}$$

in which:

$$J_1 = \frac{N + \sqrt{N^2+4}}{2} \quad \text{, always positive}$$

$$J_2 = \frac{N - \sqrt{N^2+4}}{2} \quad \text{, always negative}$$

Equation 29 is the general solution of Equation 28 and as such is applicable to both regions I and II since the differential equation describing each of these regions is given by Equation 28.

Use of the foregoing boundary conditions yields:

$$\Gamma_I = \frac{e^{J_1 \xi}}{\sqrt{N^2+4}} \;, \quad -\infty < \xi < 0 \qquad \text{............(30)}$$

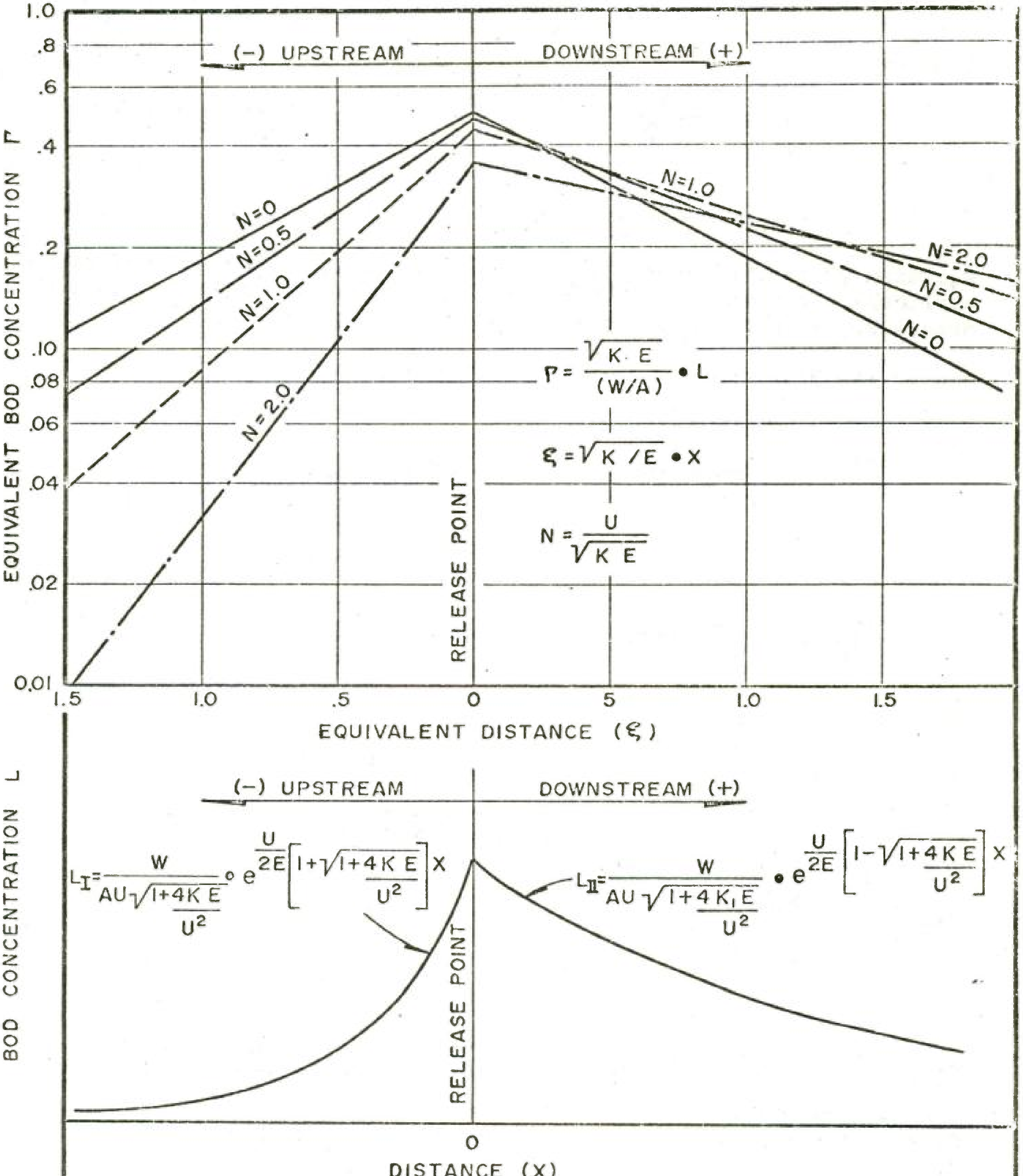
$$\Gamma_{II} = \frac{e^{J_2 \xi}}{\sqrt{N^2+4}} \;, \quad 0 < \xi < \infty \qquad \text{..............(31)}$$

In terms of system parameters, these results are:

$$\left.\begin{matrix} L_I \\ L_{II} \end{matrix}\right\} = \frac{W}{AU\sqrt{1+\frac{4KE}{U^2}}} \, e^{\frac{U}{2E}\left[1 \pm \sqrt{1+\frac{4KE}{U^2}}\right] x} \qquad \text{..........(32)}$$

Equation 32 represents the steady state distribution of BOD caused by discharge of a single plane source to a receiver of infinite extent. Graphical representation of Equation 32 and of the dimensionless profiles given by Equations 30 and 31 is given in Figure 28.

When fresh water flow has a negligible effect, Equation 32 reduces to:



BOD CONCENTRATION PROFILES INCLUDING EFFECTS OF DIFFUSION

$$\left.\begin{array}{l} L_{I} \\ L_{II} \end{array}\right\} = \frac{W}{2A\sqrt{KE}} e^{\pm\left[\sqrt{K/E}\right]x} \qquad \ldots\ldots\ldots\ldots\ldots (33)$$

This result is obtained by rearranging $U\sqrt{1+\frac{4KE}{U^2}}$ to read $\sqrt{U^2+4KE}$ and allowing $U \rightarrow 0$. The profiles for this special case are symmetrical about x=0, whereas in the general case the decay of concentration is not as rapid in the downstream region as in the upstream region. In the upstream region, the advective mechanism of transport opposes dispersion while in the downstream direction the two mechanisms reinforce one another and thus carry a relatively high concentration of waste a longer distance.

For the fresh water stream, $E > 0$ and the limiting results are:

$$\ell_{I} = 0 \qquad \ldots\ldots\ldots\ldots\ldots (34)$$

$$\ell_{II} = \frac{W}{AU} e^{-\frac{K}{U}x} \qquad \ldots\ldots\ldots\ldots\ldots (35)$$

Equation 34 is obtained by substituting E=0 in the upstream solution of Equation 32. The constant part of the exponent of e is U/O or ∞. Since this is always multiplied by a negative value of x, the resulting concentration is proportional to $e^{-\infty}$ or zero. For the case of the downstream solution, the constant part of the exponent is indeterminate, O/O. Use of L'Hôspital's rule yields Equation 35.

Equation 32 may be applied to estimate the distribution of BOD, in any of the subject waterways, caused by a point source of waste in that waterway. In particular, this equation yields a good estimate of the maximum BOD due to any given waste, provided it is some distance from the boundaries of the waterway into which it discharges. Equation 32 shows this maximum will always be located at the point of discharge.

Table 7 illustrates such an estimate for the heavy concentration of industrial waste discharging into the Arthur Kill some 4 to 6 miles below the outlet to Newark Bay. The computed value of 3.2 ppm is less than the measured values at this point of 4 to 5 ppm. This is expected since waste discharges at other points within the Kill, which will add to the concentration computed

TABLE 7

ESTIMATE OF MAXIMUM BOD CAUSED BY
POINT DISCHARGE OF WASTES
INTO ARTHUR KILL

WASTE LOAD (5 DAY BOD)	150,000 #/DAY
CENTROID OF DISCHARGE:	5 MILES SOUTH OF NEWARK BAY 8 MILES NORTH OF RARITAN BAY
LONGITUDINAL DISPERSION:	10 SQUARE MILES/DAY
CROSS-SECTIONAL AREA:	35,000 SQUARE FEET
BOD DECAY RATE:	0.5 DAY^{-1}
BOD RATIO, 5 DAY/ULTIMATE	= 0.92

$$L_{max} = L \Big|_{x=0} = \frac{W}{2A\sqrt{KE}}$$

$$= 3.2 \text{ PPM}$$

above, have not been included. The major nearby waste, not included in the computation in Table 7, is the 100,000 #/day load of the Elizabeth Joint Meeting Waste Treatment Plant. Addition of the effect of this waste would bring the BOD at the point in question into the measured range.

BOD Profile, Single Plane Source, W lbs per day, Discharging to a River Emptying into a Bay at $x=x_B$

The procedure for modifying the solution for the receiver of infinite extent, when the source of waste is close to the boundary of the waterway, is given below.

The differential equation is given by Equation 24 or 28. The boundary conditions are:

(1) $$L_{II}\Big|_{x=x_B} = L_B \qquad \text{OR} \qquad \Gamma_{II}\Big|_{\xi=\xi_B} = \Gamma_B$$

(2) $$L_I\Big|_{x=-\infty} = 0 \qquad \text{OR} \qquad \Gamma_I\Big|_{\xi=-\infty} = 0$$

(3) $$L_I\Big|_{x=0} = L_{II}\Big|_{x=0} \qquad \text{OR} \qquad \Gamma_I\Big|_{\xi=0} = \Gamma_{II}\Big|_{\xi=0}$$

(4) $$W = AE\left[\frac{dL_I}{dx} - \frac{dL_{II}}{dx}\right]_{x=0} \qquad \text{OR} \qquad 1 = \left[\frac{d\Gamma_I}{d\xi} - \frac{d\Gamma_{II}}{d\xi}\right]_{\xi=0}$$

Boundary conditions 2, 3 and 4 are identical to conditions 2, 3 and 4 for the case of the infinite receiver and have the same physical significance. Boundary condition 1 recognizes the fact that the mouth of the river is only a finite distance from the waste source. The waste concentration at the mouth or point of entry to the bay will not be zero. Assume the bay is completely mixed. The concentration at the river's mouth, L_B, must be the concentration that exists at all points in the bay. A steady state mass balance written over the bay volume yields:

$$J_i - J_o - \int_V \bar{K}\, dv = 0 \quad \text{.................... (36)}$$

in which:

J_i = mass rate of flow of waste into the bay, MT^{-1}
J_o = mass rate of flow out of the bay, MT^{-1}
$\bar{K}$ = rate of disappearance of waste by biochemical reaction within the bay, $ML^{-3}T^{-1}$

Assuming that the mass rate of inflow includes convection and dispersion, and that the rate of reaction is first order with respect to waste concentration, Equation 36 becomes:

$$-EA\frac{dL_{II}}{dx}\Big|_{x=x_B} + QL_{II}\Big|_{x=x_B} - Q\bar{L} - \int_V k\bar{L}\, dv = 0 \quad \text{........ (37)}$$

in which:

$\bar{L}$ = concentration of waste at any point in the bay, ML^{-3}
k = first order decay rate constant in the bay, T^{-1}

Since the contents of the bay are completely mixed, $L_{II}\Big|_{x=x_B}$ is equal to $\bar{L}$ and $\bar{L}$ is independent of position in the bay. Equation 37 becomes:

$$-EA\frac{dL_{II}}{dx}\Big|_{x=x_B} = kL_{II}\Big|_{x=x_B} V \quad \text{................ (38)}$$

$$\frac{d\Gamma_{II}}{d\xi}\Big|_{\xi=\xi_B} = P\Gamma_{II}\Big|_{\xi=\xi_B} \quad \text{................ (39)}$$

in which:

$P = \frac{kV}{A\sqrt{KE}}$, a second dimensionless group

The original statement of boundary condition 2 was in terms of Γ_B, an undetermined constant. Equation 39 indicates that this constant is given by

$$-\frac{1}{P}\frac{d\Gamma_{II}}{d\xi}\Big\}_{\xi=\xi_B}$$

Substitution of these boundary conditions into Equation 28 yields:

For the region upstream of the plane source -

$$\Gamma_I = \frac{1}{\sqrt{N^2+4}}\left[1-\left(\frac{P+J_2}{P-J_1}\right)e^{(J_2-J_1)\xi_B}\right]e^{J_1\xi} \qquad \text{.............(40)}$$

For the region downstream of the plane source -

$$\Gamma_{II} = \frac{(P+J_1)e^{J_2\xi}-(P+J_2)e^{(J_2-J_1)\xi_B+J_1\xi}}{\sqrt{N^2+4}\,(P+J_1)} \qquad \text{...............(41)}$$

Equations 40 and 41 reduce to Equation 32 when $\xi_B \rightarrow \infty$. Evaluation of Equation 41 at $\xi=\xi_B$ yields the concentration that will exist in the bay due to the single plane source.

$$\bar{L} = \frac{W e^{J_2\sqrt{K/E}\,x_B}}{kV+\frac{Q}{2}\left[1+\sqrt{1+4KE/U^2}\right]} \qquad \text{..........(42)}$$

When fresh water flow has a negligible influence, Equation 42 simplifies to:

$$\bar{L} = \frac{W e^{-\sqrt{K/E}\, x_B}}{kV + A\sqrt{KE}} \qquad \text{................(43)}$$

Table 8 shows the effect of this refinement on the results obtained in Table 7. The computed concentration in the bay of 0.83 ppm is more than 25% of the maximum concentration of 3.2 ppm, computed for the case of the infinite receiver. This implies that a larger and more precise result would be obtained by using Equation 41, rather than Equation 33, to compute the concentration at the point of discharge.

Dissolved Oxygen Concentration

Each BOD model has a corresponding model for the distribution of dissolved oxygen. The defining differential equation for dissolved oxygen transport in the presence of oxygen consuming organic wastes is given by Equation 5. The steady state counterpart of this equation is:

$$E\frac{d^2D}{dx^2} - U\frac{dD}{dx} - K_2 D + KL = 0 \qquad \text{..................(44)}$$

Detailed development of the steady state dissolved oxygen profiles for a single plane source of BOD in an infinite receiver is given in reference (15). The boundary conditions are:

$$D_I \Big|_{x=-\infty} \neq \infty \qquad \text{..................................BC \#1}$$

$$D_{II} \Big|_{x=\infty} \neq \infty \qquad \text{..................................BC \#2}$$

$$D_I \Big|_{x=0} = D_{II} \Big|_{x=0} \qquad \text{..........................BC \#3}$$

TABLE 8

ESTIMATE OF NEWARK BAY BOD
CAUSED BY POINT DISCHARGE OF WASTES
TO ARTHUR KILL

$$\bar{L} = \frac{We^{-(\sqrt{K/E})\,x_B}}{kV + A\sqrt{KE}}$$

W = 163,000 #/Day, Ultimate BOD

K = 0.5 Day^{-1}

k = 0.25 Day^{-1}

A = 35,000 SF

V = 0.017 Cubic Mile

E = 10 Square Miles/Day

x_B = 5 Miles

L = 0.83 PPM

$$\int_{-\infty}^{\infty} KLA\,dx = \int_{-\infty}^{\infty} K_2 DA\,dx \qquad \text{BC \#4}$$

The first two conditions state the fact that at all points, and in particular at $x = \pm\infty$, the deficit is a finite quantity. The third states that the deficit is continuous across the plane of waste discharge. The fourth states that at steady state, the oxygen consumption for waste stabilization, $\int_{-\infty}^{\infty} KLAdx$, is equal to the total oxygen transferred from the air to the water, i.e., $\int_{-\infty}^{\infty} K_2DAdx$.

Actually the fourth condition could be replaced by a statement of continuity in either the first or the second derivative at x=0. (Notice that statements of continuity in the function itself or any of its derivatives at any point within regions I or II do not constitute boundary conditions since this is information already given by the differential equation itself; i.e., the boundary conditions must yield additional information about the system.

The solutions are:

$$D_I = \left(\frac{W}{AU}\right)\left(\frac{K}{K-K_2}\right)\left[\frac{e^{\frac{U}{2E}\left[1+\sqrt{1+4K_2E/U^2}\right]x}}{\sqrt{1+4K_2E/U^2}} - \frac{e^{\frac{U}{2E}\left[1+\sqrt{1+4KE/U^2}\right]x}}{\sqrt{1+4KE/U^2}}\right] \qquad (45)$$

$$D_{II} = \left(\frac{W}{AU}\right)\left(\frac{K}{K-K_2}\right)\left[\frac{e^{\frac{U}{2E}\left[1-\sqrt{1+4K_2E/U^2}\right]x}}{\sqrt{1+4K_2E/U^2}} - \frac{e^{\frac{U}{2E}\left[1-\sqrt{1+4KE/U^2}\right]x}}{\sqrt{1+4KE/U^2}}\right] \qquad (46)$$

Determination of Critical Dissolved Oyxgen in a Receiver of Infinite Extent

The classical Streeter-Phelps development shows that the dissolved oxygen profile in a stream reaches a minimum when the decay rate is exactly equal to the reaeration rate. When longitudinal mixing is not neglected, the minimum occurs when the sum of the reaeration rate and the mixing rate, both of which supply oxygen to areas in which it is deficient, just equal the deoxygenation rate. This is expressed quantitatively by Equation 44 when dD/dx is zero. The second derivative of the deficit is negative at this point and tends to decrease just as does K_2D, the reaeration rate, whereas KL, the deoxygenation rate, is positive and tends to increase the deficit.

Details of the procedure to obtain the value and location of the maximum deficit are given in reference 15. The results, in dimensionless form, are:

$$\Gamma_{II_{MAX}} = \left(\frac{N}{N-P}\right)\left[\frac{\left(\frac{k_2\sqrt{1+4P}}{j_2\sqrt{1+4N}}\right)^{\frac{j_2}{j_2-k_2}}}{\sqrt{1+4P}} - \frac{\left(\frac{k_2\sqrt{1+4P}}{j_2\sqrt{1+4N}}\right)^{\frac{k_2}{j_2-k_2}}}{\sqrt{1+4N}}\right] \qquad \text{......(47)}$$

$$\xi_{CRIT} = \left(\frac{1}{j_2-k_2}\right) \ln\left[\frac{k_2\sqrt{1+4P}}{j_2\sqrt{1+4N}}\right] \qquad \text{..........(48)}$$

in which:

$\Gamma_{max} = \frac{Q}{W} D_{max}$

$\xi_{crit} = \frac{U}{E} x_{crit}$

$N = KE/U^2$

$P = K_2E/U^2$

$j_2 = \frac{1-\sqrt{1+4P}}{2}$

$k_2 = \frac{1-\sqrt{1+4N}}{2}$

When the velocity effect is negligible, the deficit profiles are symmetrical about the point of discharge. The maximum deficit occurs at the point of discharge and is obtained immediately from Equation 45.

$$D_{MAX} = \frac{W}{2A\sqrt{E}}\left(\frac{K}{K-K_2}\right)\left[\frac{1}{\sqrt{K_2}} - \frac{1}{\sqrt{K}}\right] \qquad \text{............ (49)}$$

Table 9 shows the evaluation of the maximum deficit in the Arthur Kill for the conditions given in Table 7. The deficit value of 3.5 ppm is lower than the observed result since neither the effect of other loads, nor the effect of the boundary at Newark Bay, are included. The requirement to assess the effect of the multi-source, multi-waterway real system leads to the construction of the working model in the next section.

TABLE 9

EVALUATION OF MAXIMUM DISSOLVED OXYGEN DEFICIT
DUE TO A POINT SOURCE OF BOD

$$D_{max} = \frac{W}{2A\sqrt{E}}\left(\frac{K}{K-K_2}\right)\left[\frac{1}{\sqrt{K_2}} - \frac{1}{\sqrt{K}}\right]$$

W = 163,000 #/Day, Ultimate BOD

A = 35,000 Square Feet

E = 10 Square Miles/Day

K = 0.5 Day^{-1}

K_2 = 0.167

D_{max} = 3.5 PPM

VI. Waste Assimilation Capacity of the Arthur Kill

Arthur Kill Mathematical Model

An early model of the New York Harbor estuarine complex considered pollutant transport in the Hudson, Harlem, East, Hackensack and Passaic Rivers, Arthur Kill and Kill Van Kull, and Newark and Upper New York Bays. The flux of BOD out of the Arthur Kill into Newark Bay was computed from the BOD profile obtained by this model and compared to fluxes in and out of the other waterways joining Newark Bay.

Figure 29 shows these results. Since the BOD concentrations in Newark Bay were larger than those in the Hackensack and Passaic Rivers, and since the convection by fresh water flow was smaller than the dispersion, 25,000 #/day of BOD was transported out of Newark Bay into these two waterways. Similarly, 16,000 #/day was transported into Newark Bay from the Kill Van Kull.

The net transport of BOD from these three waterways into Newark Bay was 9,000 #/day, less than 10% of the transport of BOD from the Arthur Kill into Newark Bay.

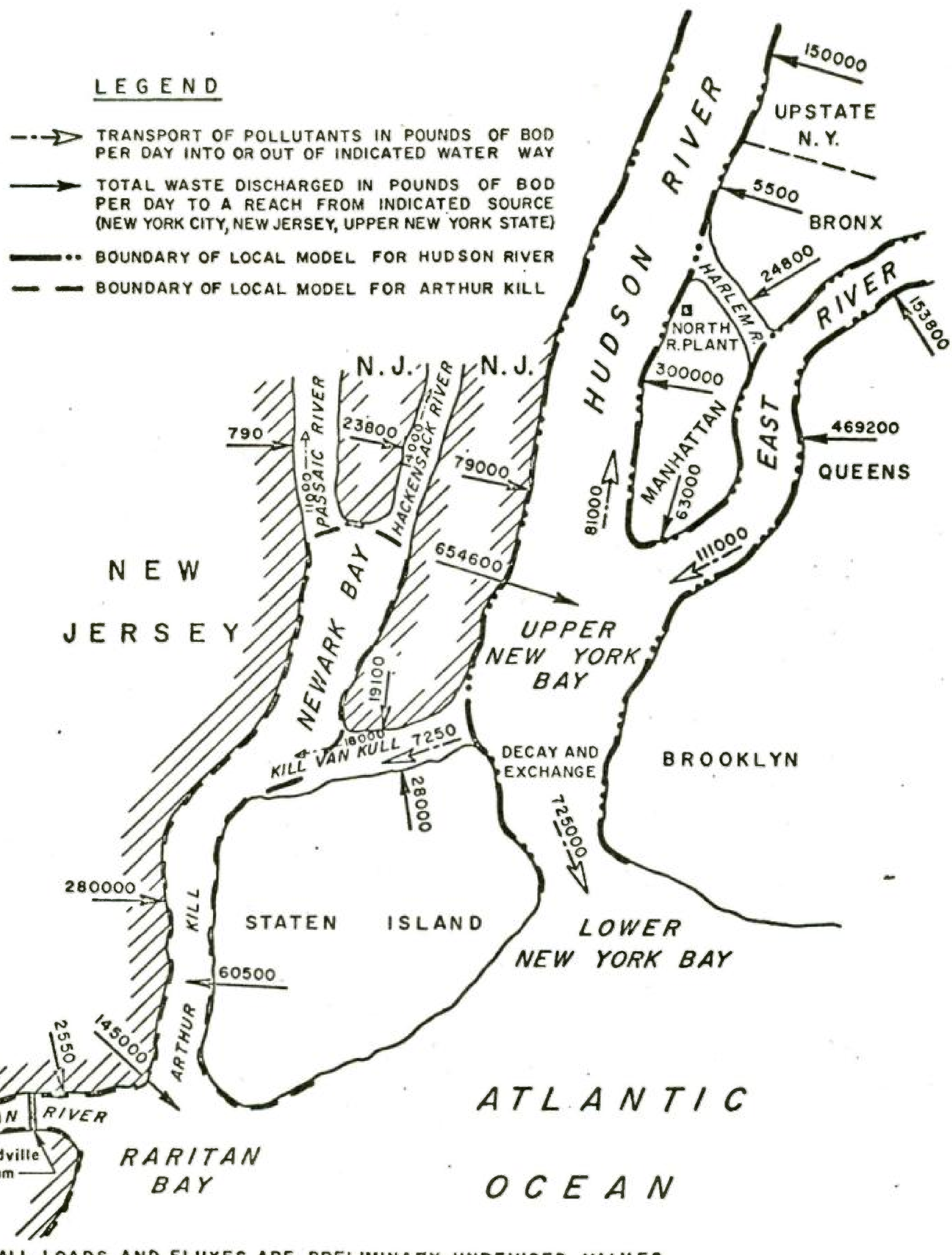
This early model neglected the Raritan River and Raritan Bay; for this reason, divergence occurred between measured and computed BOD concentrations in the southern section of the Arthur Kill.

As a result, a revised model of pollutant transport in the Arthur Kill was constructed. Since the net contribution of BOD to Newark Bay from the Hackensack, Passaic and Kill Van Kull was small by comparison to the Arthur Kill contribution, these waterways were not included in the revised model. This simpler model includes only the Arthur Kill, the Raritan River and Newark and Raritan Bays. The boundaries of both models are shown in Figure 29.

Equations 4 and 5 recognize BOD and DO concentrations depend on time and distance. Because this study considers only long term average behavior, the estuarine complex operates at steady state and time dependence vanishes. Under these conditions, as shown in Sections IV and V, these equations are second order, ordinary linear differential equations with constant coefficients; their general solutions are:

FIGURE 29

NEW YORK-NEW JERSEY HARBOR COMPLEX
PRESENT WASTE LOADINGS & POLLUTANT TRANSPORT



NOTE: ALL LOADS AND FLUXES ARE PRELIMINARY, UNREVISED VALUES

$$L_{(x)} = C_1 e^{jx} + C_2 e^{kx} \qquad \text{..........................(50)}$$

in which:

$$\left.\begin{matrix} j \\ k \end{matrix}\right\} = \frac{U}{2E}\left[1 \pm \sqrt{1 + \frac{4KE}{U^2}}\right]$$

$$D_{(x)} = T_1 e^{qx} + T_2 e^{sx} + \frac{K}{K_2 - K}\left[C_1 e^{jx} + C_2 e^{kx}\right] \qquad \text{.........(51)}$$

in which:

$$\left.\begin{matrix} q \\ s \end{matrix}\right\} = \frac{U}{2E}\left[1 \pm \sqrt{1 + \frac{4K_2E}{U^2}}\right]$$

The integration constants, C_1, C_2, T_1, T_2, are evaluated by application of suitable boundary conditions. Prior to defining these conditions, recall that Equation 4 contains no reference to the waste loads. This differential equation does not describe behavior across the plane of waste discharge. As a result, the integration constants in Equations 6 and 7 must be evaluated independently on either side of the plane of discharge; i.e., the values of C_1 and C_2, and T_1 and T_2 on the upstream side of the point of waste discharge, are not identical to their counterparts on the downstream side. Thus, for each waste discharge, four boundary conditions on BOD and four more on DO are needed to evaluate the integration constants.

This requirement of four boundary conditions each, for BOD and DO, is discussed more thoroughly in Section V. The phenomenon of multiple discharges does not seriously complicate the analysis. Since the ordinary differential equations are linear, the super-position principle applies and states that, at any point in the estuary, the overall BOD or deficit is equal to the sum of the BOD's or deficits computed for each of the individual loads. Therefore, the solution technique involves locating a single load in each reach, applying the necessary boundary conditions and computing the resultant profiles. This procedure is repeated until all loads in all reaches have been considered. The individual profiles are then added to yield the overall BOD and dissolved oxygen deficit (DOD) distributions.

Figure 30 shows the reaches above and below a source of waste in each River and the corresponding symbol for BOD and DOD. Arthur Kill parameters are designated by the subscript 1, lower Raritan River parameters by the subscript 2, upper Raritan River parameters by the subscript 3, Newark Bay parameters by the subscript 4, and Raritan Bay parameters by the subscript 5. Application of Equations 50 and 51 to the Arthur Kill yields, for the reach between a waste load and Newark Bay:

$$L_I = C_1 e^{j_1 x} + C_2 e^{k_1 x} \qquad \text{(52)}$$

$$D_I = C_5 e^{q_1 x} + C_6 e^{s_1 x} + \left[\frac{K_1}{(K_2)_1 - K_1}\right] (C_1 e^{j_1 x} + C_2 e^{k_1 x}) \qquad \text{(53)}$$

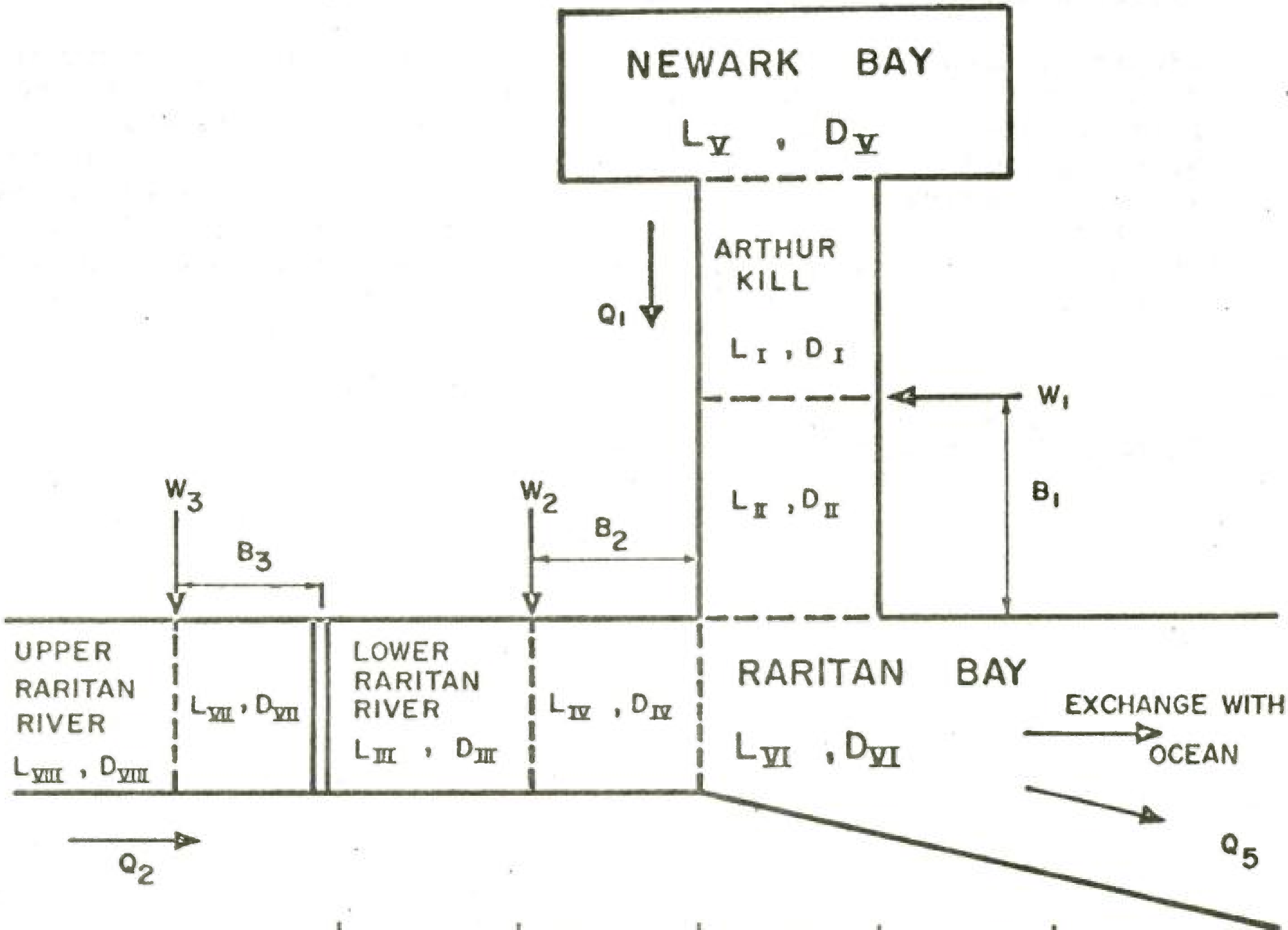
Similar application for the reach between the waste load and Raritan Bay gives:

$$L_{II} = C_3 e^{j_1 x} + C_4 e^{k_1 x} \qquad \text{(54)}$$

$$D_{II} = C_7 e^{q_1 x} + C_8 e^{s_1 x} + \left[\frac{K_1}{(K_2)_1 - K_1}\right] (C_3 e^{j_1 x} + C_4 e^{k_1 x}) \qquad \text{(55)}$$

LOCAL MODEL FOR BOD AND DISSOLVED OXYGEN TRANSPORT

ARTHUR KILL, RARITAN RIVER, & NEWARK & RARITAN BAYS



	ARTHUR KILL	LOWER RARITAN RIVER	UPPER RARITAN RIVER	NEWARK BAY	RARITAN BAY
DISPERSION COEFF.	E_1	E_2	E_3	—	—
CROSS-SECTIONAL AREA	A_1	A_2	A_3	—	—
VELOCITY	U_1	U_2	U_3	—	—
FLOW	Q_1	Q_2	Q_3	Q_1	Q_5
DEOXYGENATION COEFF	K_1	K_2	K_3	K_4	K_5
REAERATION COEFF	$(K_2)_1$	$(K_2)_2$	$(K_2)_3$	$(K_2)_4$	$(K_2)_5$
EXCHANGE COEFF.	—	—	—	—	R
VOLUME	—	—	—	V_4	V_5
WASTE LOAD	W_1	W_2	W_3	—	—
DISTANCE TO WASTE LOAD	B_1	B_2	B_3	—	—

Similar equations apply to the lower Raritan River. The Fieldville Dam acts as a natural barrier against the transport of contaminants from the lower Raritan River to the upper Raritan River; the upper Raritan River is independent of the remainder of the model, and the Streeter-Phelps analysis may be applied in this reach.

Equations 52 through 55 require eight boundary conditions for the evaluation of the eight integration constants. The similar set of equations for computing the effects of loads in the lower Raritan River require another eight conditions, which will be similar to those specified for the Arthur Kill.

A brief description of the boundary conditions required to evaluate the integration constants follows. Some of the conditions are identical to those developed in Section V.

Although Equations 4 and 5 do not define behavior in the plane of waste discharge, and discontinuity in some derivatives will occur at these points, the contaminant concentration itself is continuous and single valued, at all points.

This gives rise to the first boundary condition on BOD and DOD, i.e., only one concentration may exist in the plane of discharge or:

$$L_I \Big|_{x=0} = L_{II} \Big|_{x=0}$$

$$D_I \Big|_{x=0} = D_{II} \Big|_{x=0}$$

The second boundary condition is developed by writing a steady state material balance about the plane of discharge. In the case of BOD, this introduces the waste load into the mathematics of the model, and the material balance is constructed as follows:

$$\left[Q_I L_I - EA\frac{dL_I}{dx}\right]_{x=-\frac{\Delta x}{2}} + Q_W L_W - \left[Q_{II} L_{II} - EA\frac{dL_{II}}{dx}\right]_{x=\frac{\Delta x}{2}} = K\bar{L}A\Delta x$$

in which:

Q_W = waste flow
Q_I = net river flow above waste discharge
Q_{II} = net river flow below waste discharge
L_W = waste strength
$\bar{L}$ = average BOD within volume element

Simplifying and taking the limit as $\Delta x \rightarrow 0$ yields:

$$Q_W \left[L_W - L_I \right]_{x=0} = EA \left[\frac{dL_I}{dx} - \frac{dL_{II}}{dx} \right]_{x=0}$$

The Arthur Kill BOD, due to any load at the point of its discharge, is usually less than 1% of the BOD in the waste flow itself; thus, $L_W >> L_I$. Write $Q_W L_W$ as W, the daily waste BOD load, and obtain:

$$W = EA \left[\frac{dL_I}{dx} - \frac{dL_{II}}{dx} \right]_{x=0}$$

This is the second boundary condition on BOD.

The development of the second boundary condition on DOD follows a rationale similar to that described above and becomes:

$$\left. \frac{dD_I}{dx} \right|_{x=0} = \left. \frac{dD_{II}}{dx} \right|_{x=0}$$

These four boundary conditions apply equally to both the Arthur Kill and the Raritan River. The final four conditions for each River must be defined at the junctions at each end of each River. These are the junctions of the Arthur Kill and Newark Bay, of the upper and lower Raritan River at the Fieldville Dam, and of the two Rivers in Raritan Bay. The construction of the remaining boundary conditions begins by writing material balances for both BOD and DOD at each of the three junctions mentioned above. The procedure applied to Raritan Bay, for the case of BOD, yields:

$$+\left[Q_2 L_{IV_{B_2}} - E_2 A_2 \frac{dL_{IV_{B_2}}}{dx}\right] + \left[Q_1 L_{II_{B_1}} - E_1 A_1 \frac{dL_{II_{B_1}}}{dx}\right] - \left[Q L_{VI} + R\left(\bar{L}_{VI} - L_o\right)V_5 + K_5 \bar{L}_{VI} V_5\right] = 0$$

in which:

B_2 = distance between the waste discharge in the Raritan River and Raritan Bay, L
B_1 = distance between the waste discharge in the Arthur Kill and Raritan Bay, L
L_{VI} = average BOD concentration, Raritan Bay, ML^{-3}
Lo = average BOD concentration, Atlantic Ocean, ML^{-3}
R = coefficient of tidal exchange between Raritan Bay and Atlantic Ocean, T^{-1}
V_5 = volume of Raritan Bay, L^3
Subscript 1 refers to Arthur Kill
Subscript 2 refers to Raritan River
IV is the region downstream of waste discharge in Raritan River
II is the region downstream of waste discharge in Arthur Kill

The first two bracketed terms represent transport of BOD from the Raritan River and Arthur Kill into the Bay. The third term represents transport of material out of the Bay by net flow to the Atlantic Ocean. The fourth term defines the rate at which BOD in the Bay, L_{VI}, is exchanged with ocean BOD, L_o, due to tidal flushing. R is the tidal exchange coefficient; for the case of zero ocean BOD, R represents the fraction of BOD flushed from the Bay per day. The final term delineates first order decay within the Bay.

A similar material balance may be written for DOD at this junction and for BOD and DOD at the other two junctions. For Newark Bay, no tidal exchange has been used. At the Fieldville Dam, transport of material is continuous across the dam, there is no dispersion in the upper Raritan, and the decay and exchange terms vanish since the junction volume is zero.

Thus, six additional equations have been introduced toward the evaluation of the remaining eight integration constants. However, these equations themselves have introduced additional constants. For example, the material balance above introduced two new constants, namely L_{VI} and L_O, the Raritan Bay and ocean BOD concentrations, respectively. The deficit equation at this junction will introduce D_{VI} and D_O.

At Newark Bay, the BOD and DOD concentration in Newark Bay is introduced and, at the Fieldville Dam, the upstream BOD and DOD concentrations appear. In all, eight new constants, in addition to the original sixteen, must now be evaluated. Since only fourteen conditions have been introduced at this point, another ten conditions must be specified to complete evaluation of the twenty-four constants.

The ocean BOD and deficit, L_O and D_O are assumed to be zero. Both bays are considered to be completely mixed. At a river-bay junction, therefore, the river concentration is equal to the bay concentration. At the junction of Raritan Bay, Raritan River and Arthur Kill, this introduces two equations for BOD and two for DOD, i.e.:

$$L_{II}\Big|_{x=B_1} = L_{VI}$$

$$L_{IV}\Big|_{x=B_2} = L_{VI}$$

$$D_{II}\Big|_{x=B_1} = D_{VI}$$

$$D_{IV}\Big|_{x=B_2} = D_{VI}$$

Similarly, at the junction of Arthur Kill and Newark Bay, one equation each is written for BOD and DOD.

The upper Raritan River BOD and DOD equations, which appear in the expression for the continuity of transport across the Dam, are obtained by application of the Streeter-Phelps equations for BOD and DOD in the upper River.

These ten statements constitute the additional ten required boundary conditions. Table 10 summarizes these generated constants and the boundary conditions required for their evaluation.

Four of the twenty-four constants (the ocean and upper Raritan River concentrations) are evaluated independently. The remaining twenty are evaluated by solution of a set of twenty simultaneous equations; i.e., the twenty remaining boundary conditions.

The resulting 20 by 20 matrix was solved on the computer by a standard matrix inversion technique. After determining the constants of integration, the program utilized Equations 52 through 55 to compute the BOD and DOD for each load. Summation then gave the cumulative BOD and DO distributions for the total load on the estuary.

Arthur Kill Model Verification

Model verification proceeds by comparing machine computed BOD and DO distributions to measured values of BOD and DO. Values of River Parameters and of Existing Loads, as developed in Section IV, are the computer input. Measured values of BOD and DO in the Arthur Kill were obtained from NYC Department of Public Works Harbor Survey Reports, June to September 1964, and from the ISC Arthur Kill Survey, June 17 to July 18, 1957. Mean NYCDPW values represent an average of 24 values at each sampling point, i.e., top and bottom samples taken once a week for 12 weeks. Mean ISC values represent an average of about 50 values at each sampling point, i.e., one sample taken five feet below the surface three times a day, four days a week, for four weeks.

Computed results for Arthur Kill BOD, DOD and DO for existing loading conditions are presented in Table 11. A comparison of measured and computed values of BOD and DO is shown in Figure 31.

ISC dissolved oxygen data in Figure 31 represents an average condition for the four week period, June 17 to July 18, 1957. Samples were collected at a single point at each station. Since the Kill is relatively well mixed laterally, these data probably

TABLE 10

GENERATED CONSTANTS AND AVAILABLE BOUNDARY CONDITIONS

CONSTANTS		AVAILABLE BOUNDARY CONDITIONS		
	Generated by General Solution of Differential Equation for Each River	At Point of Load Discharge		
Arthur Kill BOD	4	2		
Arthur Kill DOD	4	2		
Lower Raritan River BOD	4	2		
Lower Raritan River DOD	4	2		
	Generated by Material Balance at Each Junction	Junction Balance	Complete Mix Assumption	Concentration Specification
Arthur Kill-Newark Bay BOD	1	1	1	---
Arthur Kill-Newark Bay DOD	1	1	1	---
Arthur Kill-Raritan River-Raritan Bay BOD	2	1	2	1
Arthur Kill-Raritan River-Raritan Bay DOD	2	1	2	1
Upper & Lower Raritan River BOD	1	1	---	1
Upper & Lower Raritan River DOD	1	1	---	1

Total Constants for Evaluation 24

Total Available Boundary Conditions 24

TABLE 11

COMPUTER OUTPUT FOR
ARTHUR KILL BOD AND DO PROGRAM (RRAKDO)
UNDER EXISTING LOADING CONDITIONS

Distance (Miles)*	BOD (PPM)	Deficit (PPM)	DO (PPM)	Per Cent Saturation
0	1.9046	3.3299	4.6701	.5838
1	1.8794	4.0451	3.9549	.4944
2	1.9486	4.6891	3.3109	.4139
3	2.1156	5.2674	2.7326	.3416
4	2.3736	5.7775	2.2225	.2778
5	2.703	6.2088	1.7912	.2239
6	3.0588	6.5445	1.4555	.1819
7	3.5018	6.7636	1.2364	.1546
8	4.0536	6.837	1.163	.1454
9	4.5111	6.7277	1.2723	.159
10	4.5976	6.402	1.598	.1997
11	4.3084	5.8479	2.1521	.269
12	3.3492	5.0838	2.9162	.3645
13	2.5146	4.1571	3.8429	.4804

* Measured from Raritan Bay

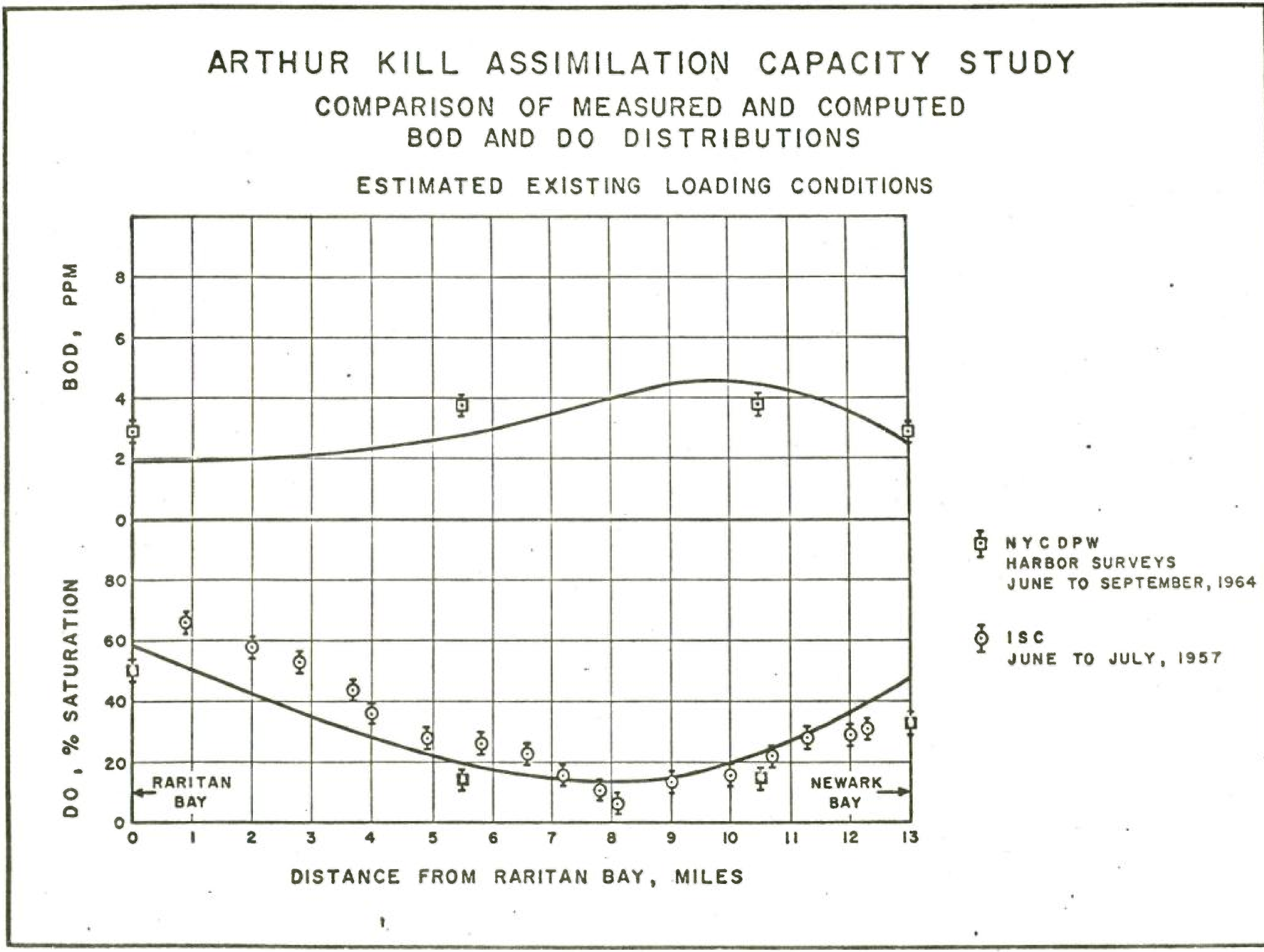

ARTHUR KILL ASSIMILATION CAPACITY STUDY
COMPARISON OF MEASURED AND COMPUTED
BOD AND DO DISTRIBUTIONS
ESTIMATED EXISTING LOADING CONDITIONS
BOD, PPM
DO, % SATURATION
RARITAN BAY
NEWARK BAY
DISTANCE FROM RARITAN BAY, MILES
NYC DPW
HARBOR SURVEYS
JUNE TO SEPTEMBER, 1964
ISC
JUNE TO JULY, 1957

FIGURE 31

represent a good estimate of the cross-sectional area average. The DO's in the southern third of the Kill for this survey are higher than the computed values because the Middlesex County Sewage Treatment Plant, which contributes 150,000 #/day BOD to Raritan Bay, was not on line at that time. Recognizing this, the ISC data support the trend displayed by the computed profiles.

Loading conditions prevailing during the 1964 NYCDPW surveys more closely represent the conditions simulated by the model. These data were collected during June through September 1964, and the averages over top and bottom and over several weeks are good estimates of the tidal-smoothed, area-averaged concentrations. These data support the trends shown by the computed BOD and DO profiles.

Although the shape of the computed DO curve in Figure 31 parallels movement of NYCDPW DO values, the computed DO is about 10% higher than these measured values. Arthur Kill New Jersey loads, given in Table 2, were for the most part computed from grab samples, and may deviate from long term average conditions. These are the known outfalls; unknown loads will increase the total Kill load and decrease the computed DO. Benthal demand is not considered in the model. Sludge deposits are extensive in the Kill and, if considered, would reduce computed oxygen levels.

For these reasons, the New Jersey loads shown in Table 2 were increased by 10% to reflect a more realistic Kill loading. Figure 32 compares computed and measured DO values for the increased loading conditions. The computed DO is 6 and 11% saturation units higher than the measured value at Raritan and Newark Bay, respectively.

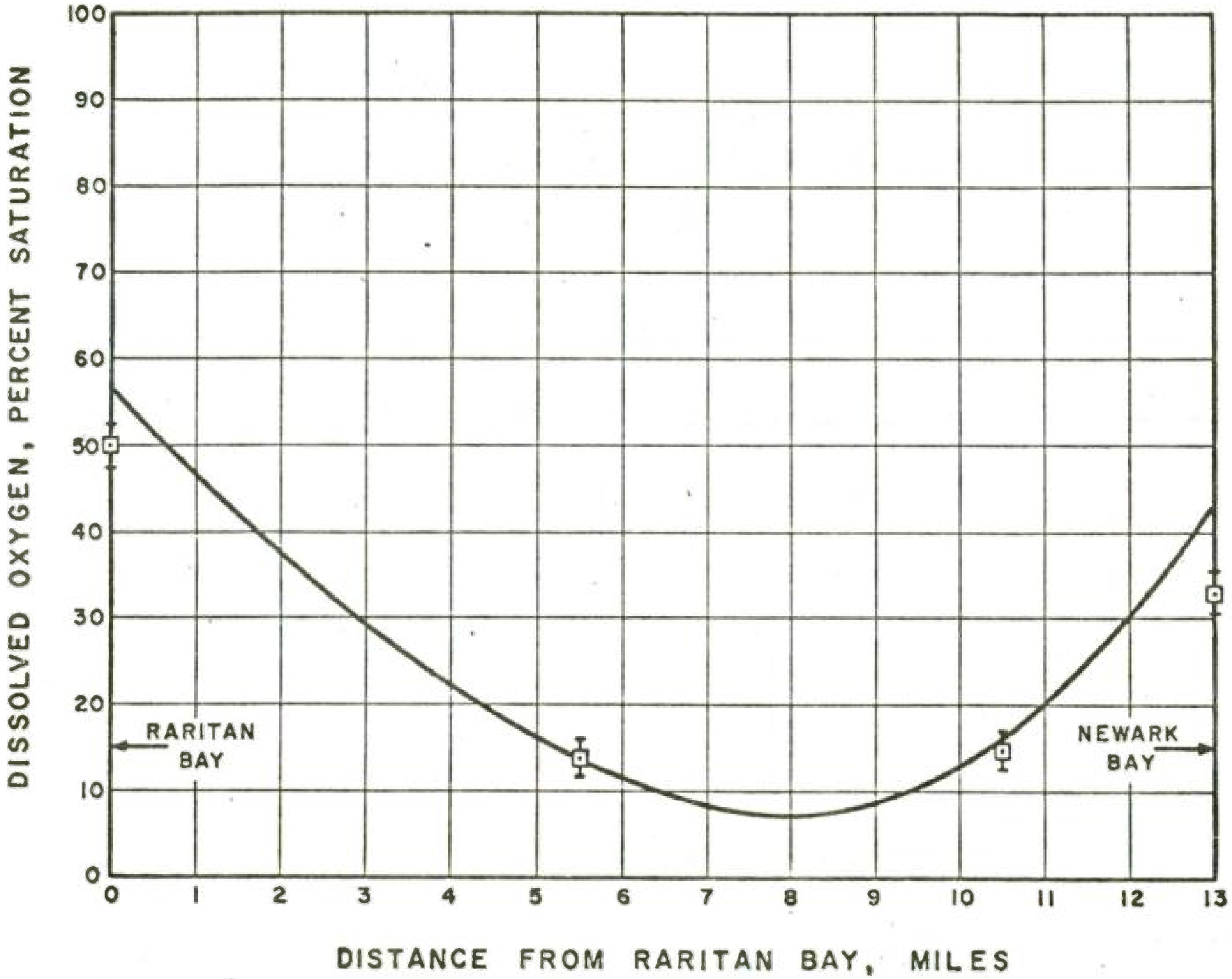
These boundary inconsistencies may be due to model simplifications or to presence of unaccounted Bay loadings. Elimination of Kill Van Kull and the Hackensack and Passaic Rivers may lower the total load on Newark Bay. Limitation of the fraction of Raritan Bay volume subject to pollution provides an artificially high volume of unpolluted ocean water available for tidal exchange.

Figure 32 shows the good agreement between measured and computed DO concentrations in the central region of the Kill and demonstrates accurate model prediction of minimum average dissolved oxygen levels Stream standards of all regulatory agencies exercising control in the Arthur Kill are directed toward maintaining DO concentrations above prescribed minima. The model may be used to guide the increase in waste treatment to achieve these prescribed minima.

ARTHUR KILL ASSIMILATION CAPACITY STUDY

COMPARISON OF MEASURED VALUES TO THE COMPUTED DISSOLVED OXYGEN DISTRIBUTION

REVISED LOADING CONDITIONS



AVERAGE OF MEASURED VALUES NYCDPW HARBOR SURVEYS JUNE TO SEPTEMBER, 1964

$T = 70°F$

$C_s = 8.0$ MG/LITER

$K_2 = 0.167$ DAY^{-1}

$K_1 = 0.5$ DAY^{-1}

$E = 10$ SQUARE MILES/DAY

$U = 0.0$

$A = 0.00125$ SQUARE MILES

FIGURE 32

Effect of Individual Waste Discharges

The total load discharged to Raritan Bay is 162,000 #BOD/day. This load is discharged by the Middlesex County Sewerage Authority, Perth Amboy and South Amboy. Table 2 lists the individual waste discharges to the Bay.

Figure 33 shows the DO depletion caused by this load by comparison to total load. At the point of minimum DO (mile point 8), the Raritan Bay load causes a depletion of 12% of saturation; this represents 13% of the total depletion at this point. Average depletion throughout the reach due to this load is 14% of saturation.

This is an exaggerated effect because the mathematical model treats the total load as being discharged at the mouth of the Arthur Kill.

In reality, the load is discharged farther out in Raritan Bay. This analysis demonstrates, however, that the load on Raritan Bay, approximately 30% of the total load on the Arthur Kill, does not contribute proportionately to the large depletions at the critical point.

A total New Jersey waste load of 164,000 #BOD/day is discharged to the Arthur Kill between mile points 7.75 and 9.8 above Raritan Bay. Table 2 lists the individual discharges that compose this total load.

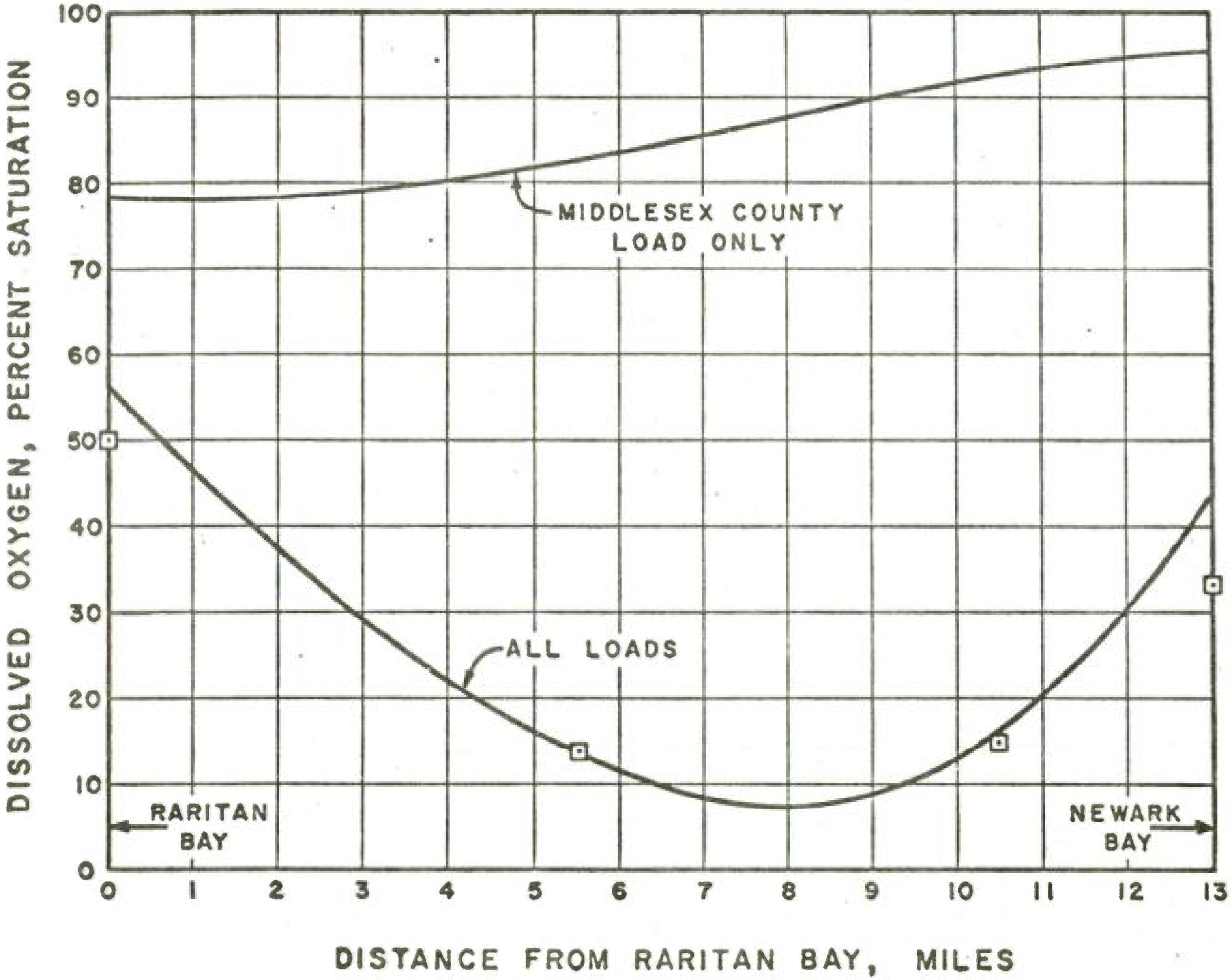
Oxygen depletion caused by this load by comparison to total depletion is shown in Figure 34. This load, which is about 30% of total Kill load, causes a depletion of 42 percentage saturation units at the critical point. This corresponds to 46% of total depletion at the minimum point.

Prediction of Water Quality after Increased Treatment

The amount of treatment required to meet stream standards may be estimated from the model by decreasing the loads (increasing waste treatment) and computing a corresponding minimum DO. The model was verified for three-month summer averages and computed concentrations under any loading condition are assumed to represent the three-month (quarterly) summer average that would occur under that loading condition.

ARTHUR KILL ASSIMILATION CAPACITY STUDY

RELATIVE EFFECT OF INDIVIDUAL AND OVERALL LOADING ON COMPUTED DISSOLVED OXYGEN DISTRIBUTION



⊡ AVERAGE OF MEASURED VALUES NYC DPW HARBOR SURVEYS JUNE TO SEPTEMBER, 1964

$T = 70°F$

$C_s = 8.0$ MG/LITER

$K_2 = 0.167$ DAY^{-1}

$K_1 = 0.5$ DAY^{-1}

$E = 10$ SQUARE MILES/DAY

$U = 0$

$A = 0.00125$ SQUARE MILES

FIGURE 33

ARTHUR KILL ASSIMILATION CAPACITY STUDY

RELATIVE EFFECT OF INDIVIDUAL AND OVERALL LOADING ON COMPUTED DISSOLVED OXYGEN DISTRIBUTION

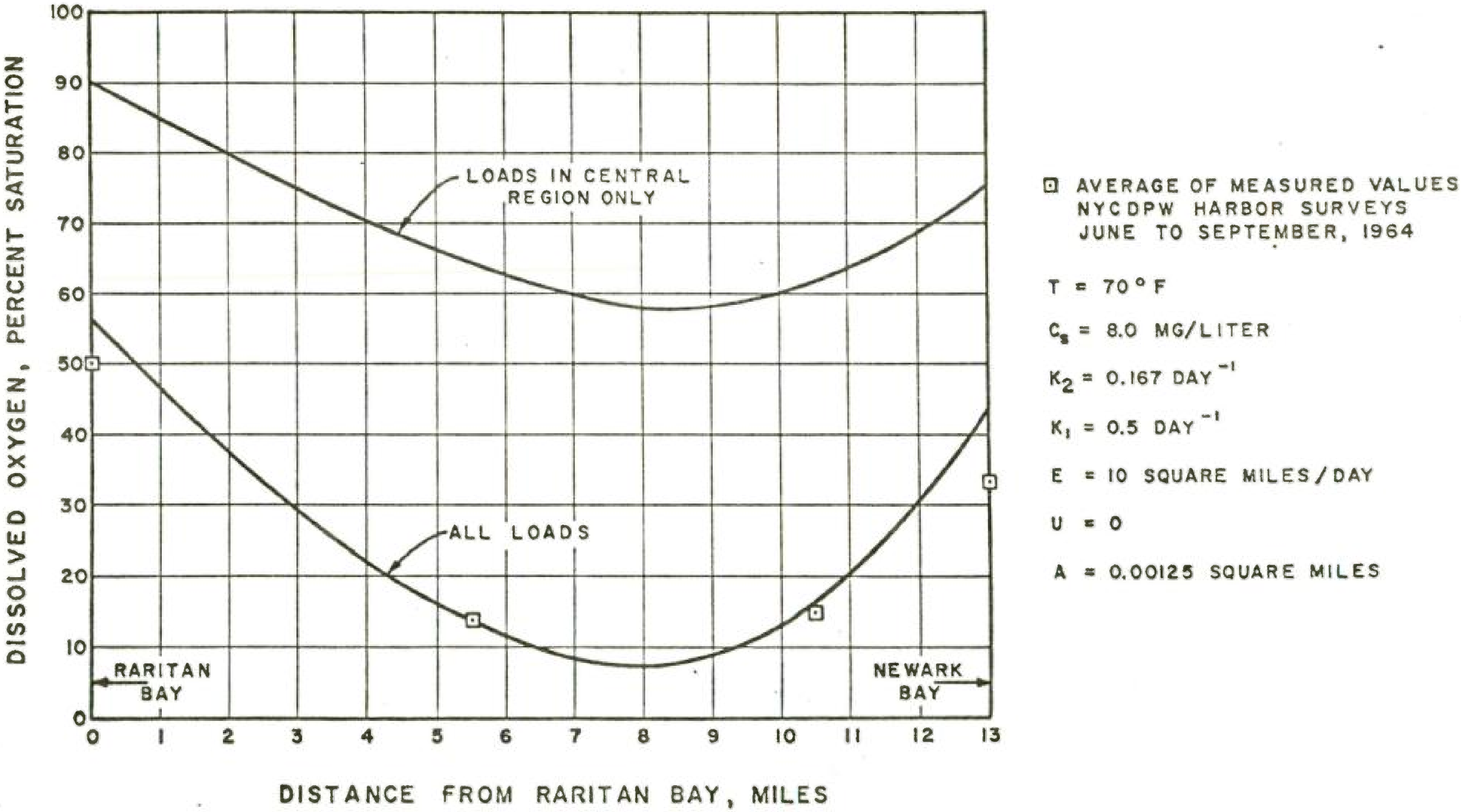


FIGURE 34

Average dissolved oxygen levels for shorter periods will fluctuate above and below the long term quarterly average. Measured variations about the three-month summer average were regarded as characteristic of the variations that will occur under future conditions of reduced load. To provide protection for the short term, and to approximate meeting the standards which are written on an "at any time" basis, waste treatment requirements were based on a minimum quarterly dissolved oxygen concentration which exceeded the NJSDH standard of 30% dissolved oxygen saturation at any time by the observed variation between the quarterly average and the short term value.

Variation below the three-month summer average during 1964 harbor survey is shown in Table 12. A maximum variation of 21% of saturation occurred. The remainder of the variations were either equal to or less than 20% of saturation. On this basis, a value of 20% of saturation was used, in computing treatment requirements, to represent the difference between the minimum three-month average value and the 30% standard.

Waste treatment requirements in the Arthur Kill currently include 80% influent BOD removal. In Table 2, some of the loads represent no treatment at all, whereas data on those treatment plants which do exist on the Kill show BOD removals as high as 40%. One equitable treatment regulation requires that percentage BOD removal be equal for all waste sources. Thus, treatment computations must be made on the basis of present influent or raw loads.

Sufficient data were not available to establish long term average BOD removals at existing treatment plants. Computations for required treatment were made by assuming all loads given in Table 2 provide the same level of treatment. These levels were chosen to range from 0 to 40% BOD removal.

Table 13 lists the percentage treatment estimates necessary to meet stream standards for various initial BOD removals. Figure 35 shows the influence of BOD removal on minimum dissolved oxygen levels in the Arthur Kill. BOD removal for the condition of zero present treatment represents that which must be provided should no credit be given to existing levels of treatment, i.e., to maintain the 30% saturation level at all times, each existing load requires 53% removal.

Treatment of waste sources on the Kill probably ranges between 15 and 25%. Should a 15% credit be applied for existing treatment, influent loads would require 60% treatment to maintain the minimum standard. This does not change the quantity of material discharged

TABLE 13

BOD REMOVAL TO OBTAIN STREAM STANDARDS
FOR DIFFERENT INITIAL BOD REMOVALS

Present BOD Removal	Total BOD Removal Required
0	53
15	60
25	65
40	71

TABLE 12

VARIATION OF 1964 THREE MONTH SUMMER AVERAGE DO
WITH THE MINIMUM DO IN THE ARTHUR KILL

Station	Average DO % Saturation	Minimum DO % Saturation	Variation % Saturation
Shooters Island	33	12	21
B & O Bridge	15	0	15
Fresh Kills	14	4	10
Tottenville	51	32	19
Raritan River	50	30	20

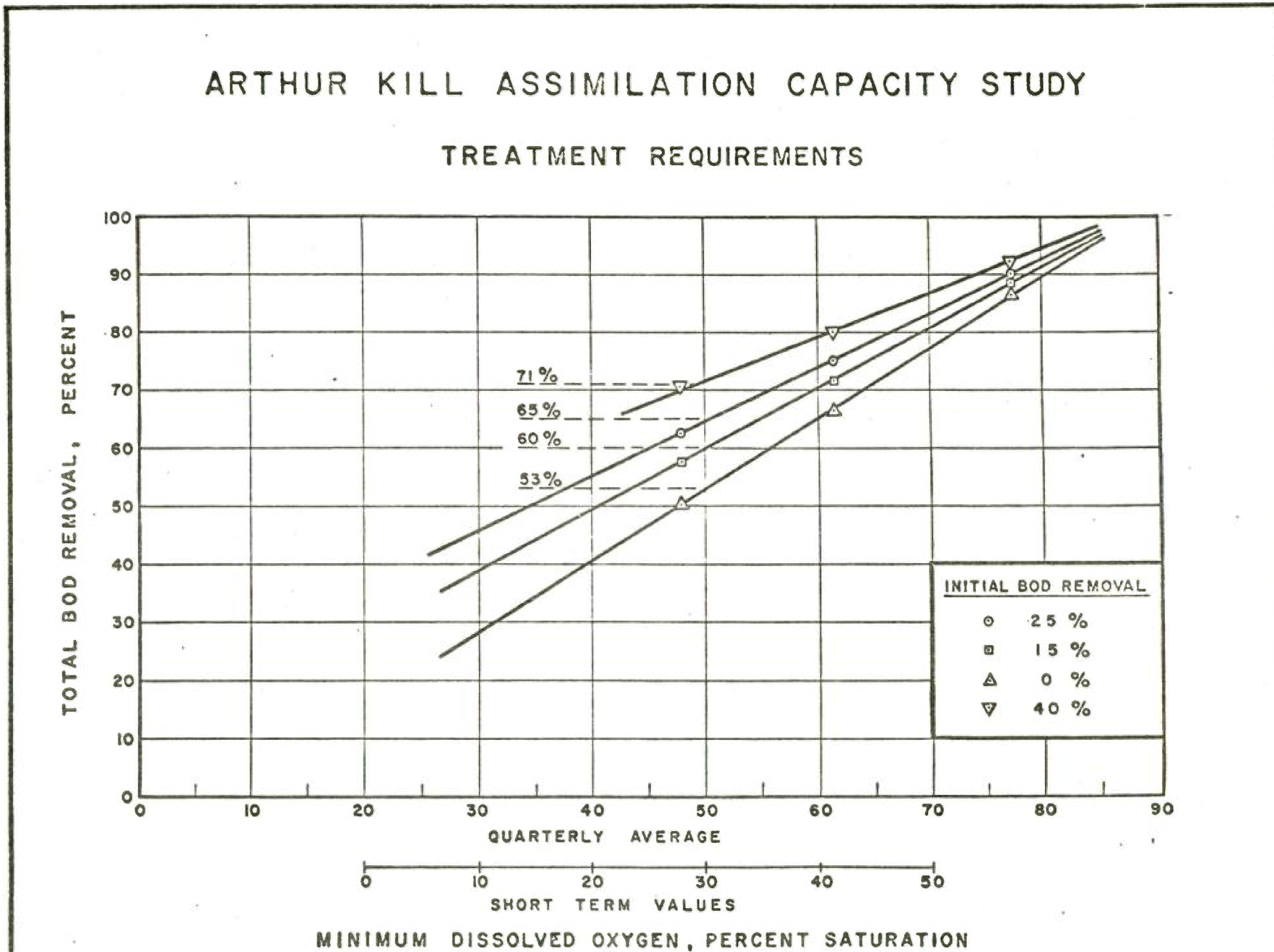

ARTHUR KILL ASSIMILATION CAPACITY STUDY
TREATMENT REQUIREMENTS
TOTAL BOD REMOVAL, PERCENT
100
90
80
70
60
50
40
30
20
10
0
71 %
65 %
60 %
53 %
INITIAL BOD REMOVAL
25 %
15 %
0 %
40 %
0
10
20
30
40
50
60
70
80
90
QUARTERLY AVERAGE
0
10
20
30
40
50
SHORT TERM VALUES
MINIMUM DISSOLVED OXYGEN, PERCENT SATURATION

FIGURE 35

to the Kill. For example, the condition of zero present treatment permits 47% of the present load to be discharged to the Kill. For the case of a 15% present treatment credit, influent load is 118% (100/0.85) of the present load on the Kill. Sixty per cent removal of this influent load will leave 40% of the influent to be discharged to the Kill. Forty per cent of this influent is equivalent to 47% of the present effluent loading.

Table 13 shows, for the loading estimate given in Table 2 (increased by 10%), the estimated required treatment ranges between 50 and 75%, and depends on the treatment credit applied to the existing loading level. Present treatment levels on the Kill should be established. Once these data are available, this computer model may be used to estimate overall treatment requirements of present and future influent loads, required to maintain the 30% standard.

Assimilation Capacity and Treatment Requirements

This section considers levels of treatment for upgraded standards in the Arthur Kill. The assimilation capacity of the Kill for several levels of prevailing temperatures is estimated. Allocations of the assimilation capacity for future growth and between the States of New York and New Jersey are considered. Very preliminary estimates of the nitrogeneous effect on the capacity and of the equivalent BOD of thermal discharges are made.

Figures 36 and 37 show the percentage waste treatment required in the Kill, for various levels of temperature, to meet a given quarterly dissolved oxygen minimum. Figure 36 presents the minimum in absolute dissolved oxygen concentration units while, in Figure 37 the percentage saturation of dissolved oxygen appears as the quarterly minimum. Dissolved oxygen saturation was obtained from reference 13 for the prevailing temperature and a chloride concentration of 10,000 ppm.

Loading conditions for these curves are the original loading estimates presented in Table 2, rather than the 10% across the board loading increase applied in the previous section. This was done because the original total loading to all waterways contained in the model of 500,000 #/day BOD agreed* closely with the FWPCA (16) total loading estimate of 495,000 #/day BOD. Percentage

*The loading distribution among the model waterways did not agree. FWPCA estimates were lower for the Arthur Kill and higher for the Raritan River. This may be due in part to arbitrary location of loads at the junction of these two waterways. Agreement between Raritan Bay estimates was better.

ARTHUR KILL ASSIMILATION CAPACITY

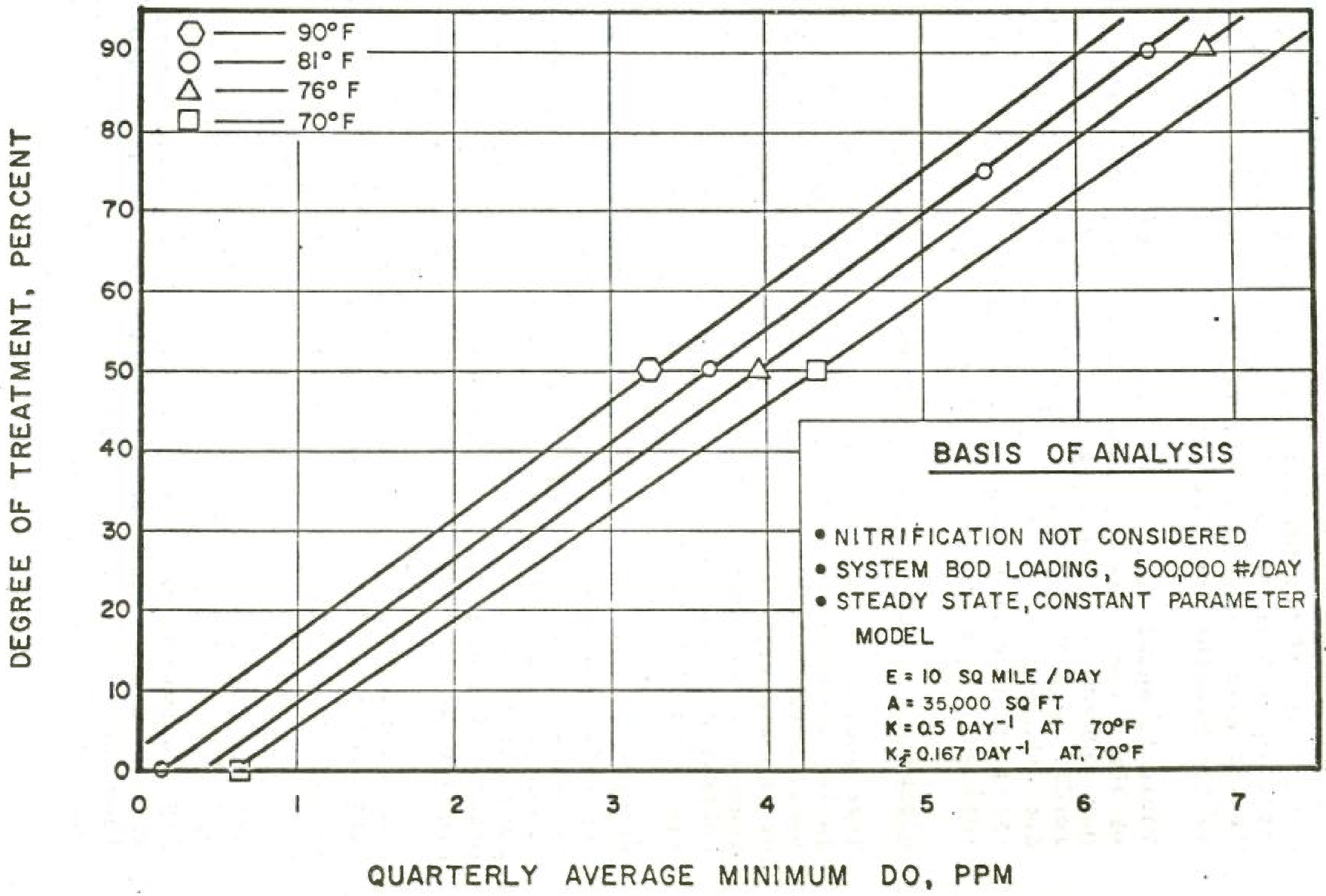


FIGURE 36

ARTHUR KILL ASSIMILATION CAPACITY

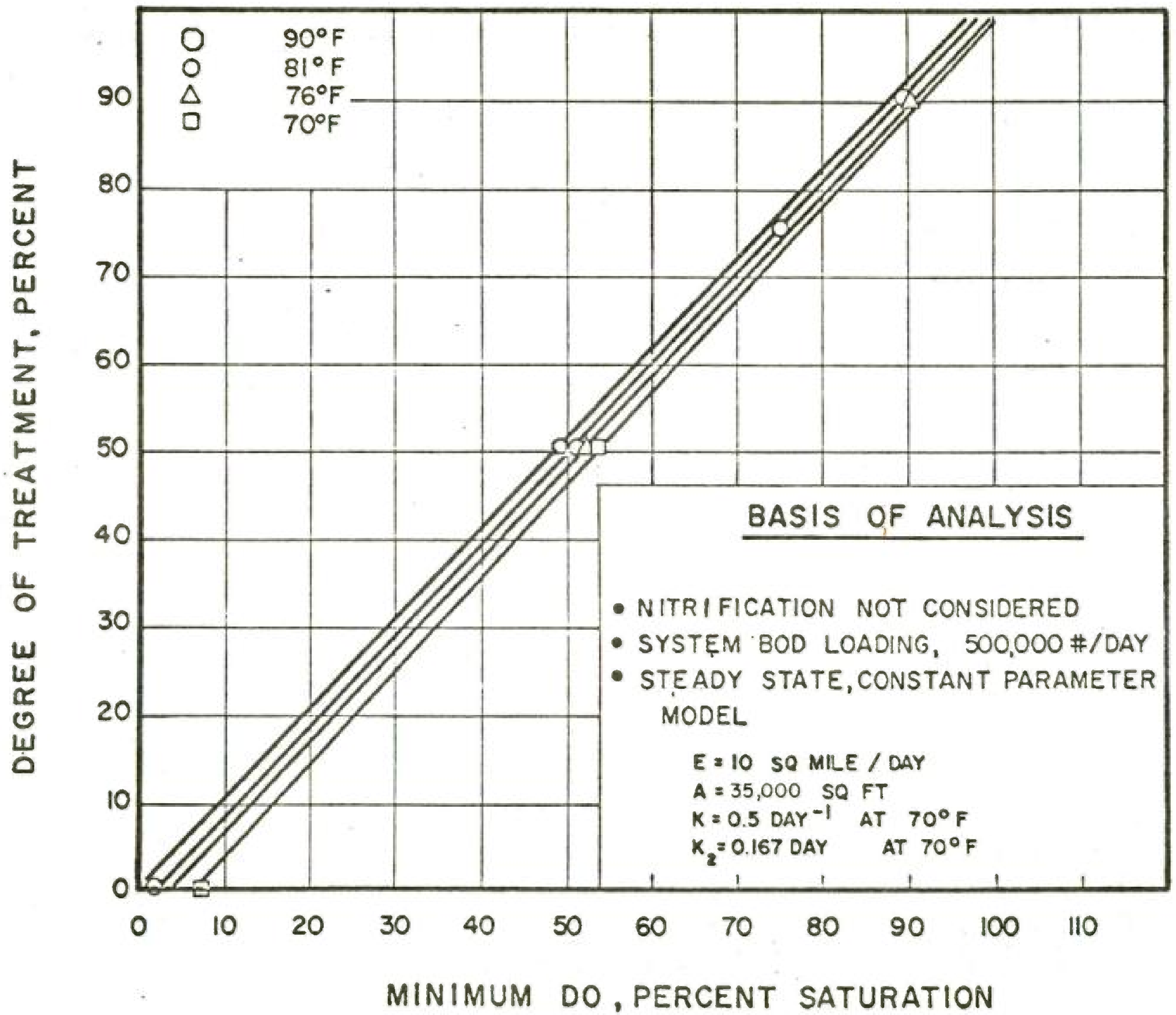


FIGURE 37

treatment estimates are based on these loads directly; i.e., they were not computed on the basis of influent loads and existing levels of treatment, as done in the previous section.

Figures 36 and 37 are based directly on model computed minimum DO values and do not differentiate between average, longer term values and the values "at any time." As discussed previously, these model computed values are an accurate representation of average, longer term conditions, but yield DO values that are 20% higher than the minimum observed "point" values at the present time.

Computation of increased % treatment requirements in this section was based on the long term averages because the variance between average and point values in the presence of treatment is not known. This variance is due in part to tidal cycle variation and in part to fluctuations in untreated waste discharges. A marked reduction in the latter could be achieved by treatment of all sources.* Additional investigation of this variation of DO about a mean and expected performance after treatment is needed.

These Figures may be used to estimate the effect of upgraded standards. A potential DO standard of 4 ppm in the Kill is discussed; regulatory agencies have discussed this as an average requirement, to be accompanied by a 2.5 to 3.0 ppm "at any time" value. These limits exceed the present observed variation of 20% discussed above, so that the 4.0 ppm average would be expected to be the control, particularly in the presence of treatment.

Figure 36 shows an overall treatment requirement of 55% in the Arthur Kill at $81^{\circ}F$. This temperature has been used because heated effluents discharged to the Kill have been estimated to have raised the Kill temperature about $5^{\circ}F$ above the ambient in the summer. A temperature of $76^{\circ}F$ is representative of average summer ambient conditions in the subject waterways. Based on a loading of 500,000 #/day, this treatment requirement means that the assimilation capacity of the Kill is 225,000 #/day; i.e., at $81^{\circ}F$, the Kill will assimilate 225,000 #/day BOD without dropping below the 4.0 ppm minimum.

If one-half of this available capacity is allocated to New York State and one-half to New Jersey, each State could permit users within its jurisdiction to discharge a total of 112,500 #/day BOD to the Kill. Table 2 shows that present New York loading sources

*Some slug waste loads will continue to be discharged since cooling water flows from many industries can become contaminated. For example, leaky shell and tube heat exchangers can waste product to the cooling water side.

total 60,000 #/day while New Jersey sources total 440,000 #/day. To limit its use to 112,000 #/day, New Jersey must require 75% removal of present loads. New York loading is less than its assimilation capacity allocation. Removal of 75% of the 60,000 #/day New York load would leave a reserve capacity of 97,000 #/day.

Should treatment of all existing effluent loads meet the ISC requirement of 80% treatment, a total of 100,000 #/day BOD would be discharged to the Arthur Kill-Raritan complex. This would provide a reserve capacity of 125,000 #/day for future loads. On an equal allocation basis, New Jersey would be utilizing 88,000 #/day of its capacity allocation of 112,000 #/day and New York, 12,000 #/day of its 112,000 #/day.

Considering the largely industrialized New Jersey side of the Kill versus the residential New York side, 50-50 allocation of the assimilation capacity may be subject to question. However, on the quarterly average basis, a uniform treatment requirement of 80% for all users will provide reserve capacity equal to 55% of the total Kill capacity. At the 80% treatment level, this reserve capacity of 125,000 #/day would mean future additional influent loadings of 625,000 #/day (425,000 #/day to the Kill, 200,000 #/day to the Raritan) could be handled without increasing the treatment level.

This future additional loading is 25% greater than the present loading on the subject waterways. A treatment requirement of 90% of present loads would leave a reserve capacity of 175,000 #/day and would permit a future additional loading of 1,750,000 #/day. Although it is difficult to project future industrial waste loads, it is unlikely that the additional future loads will be three and one-half times the present load. At this time, it appears that the 80% treatment requirement will be sufficient to elevate the minimum Arthur Kill dissolved oxygen content to 4.0 ppm.

Effect of Thermal Effluents

A very preliminary estimate of the effect of heated effluents on the assimilation capacity of the Arthur Kill complex is made by comparing the assimilation capacity of the Kill at $76^{\circ}F$ to that at $81^{\circ}F$. The $5^{\circ}F$ change represents a preliminary evaluation of New York-New Jersey waterway temperatures and is roughly supported by calculations shown in Table 14.* These calculations are based on a model constructed from the energy balance analog of Equation 2. Boundary conditions are for the single source of waste (heat) into a receiver of infinite extent. The equation used is the temperature analog of Equation 33.

From Figure 36, the treatment requirement at $76^{O}F$ and a 4.0 ppm DO standard is 51%; the assimilation capacity is, therefore, 245,000 #/day. This is 20,000 #/day more than the capacity at $81^{O}F$ and, at 80% treatment, represents an additional 100,000 #/day influent BOD.

A comparison of treatment plant and cooling tower costs shows that the cost of treatment of 100,000 #/day is roughly comparable to the cost of cooling towers for the 200 billion BTU/day of waste heat estimated by reference (16) to be discharged daily to the Arthur Kill.

Since the cost of correcting the approximate loss of 100,000 #/day of assimilation capacity, caused by the waste heat discharge, appears to be roughly comparable for either treatment of an additional 100,000 #/day or for installation of cooling towers, the concept of requiring heat removal from cooling water merits additional investigation. Cooling water control would provide additional control on intermittent waste discharge through cooling water channels.

Effect of Nitrogeneous Oxygen Demand

The oxygen deficit due to nitrogeneous oxygen demand was found to vary between 1.3 and 2.3 ppm in the Thames Estuary (17). Total oxygen deficit was of the order observed in the Arthur Kill.

In the Arthur Kill, dissolved oxygen saturation at $81^{O}F$ is 7.14 ppm, leaving a maximum deficit of 3.14 ppm corresponding to the 225,000 #/day assimilation capacity. If 1.3 ppm of this is set aside for meeting the expected nitrogeneous demand, 93,000 #/day of nitrogeneous assimilation capacity is provided. The leaves 132,000 #/day for assimilation of the effluent carbonaceous BOD.

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*This preliminary model assumes a centrally located single load in an unbounded waterway and yields a temperature range of $7.9^{O}F$ at the plane of discharge to $3.9^{O}F$ at the entrance to either bay. Consideration of the bounding effects of both bays, and a more detailed investigation of the magnitude and distribution of the thermal effluents, of the meterological conditions controlling the heat transfer coefficient, and of the Kill temperature distribution, would be required to improve the estimate of the thermal effect.

TABLE 14

ESTIMATION OF EFFECT OF
HEATED EFFLUENTS ON
ARTHUR KILL TEMPERATURE

DEFINING EQUATION: $\frac{Ed^2\Delta T}{dx^2} - \frac{Ud\Delta T}{dx} - K_T\Delta T = 0$

CONDITIONS: Single Plane Source, Infinite Receiver, No Velocity

DISTRIBUTION EQUATION: $\Delta T = \frac{H\, e^{-\sqrt{K_T/E}\,|x|}}{2\rho C_p A\sqrt{K_T E}}$

H = 200 Billion BTU/Day

K_T = 0.12 Day^{-1} (estimated from heat budget and prevailing meteorlogy. A thermal stratification factor of 2.0 is used*

E = 10 Square Miles/Day = 2.8×10^8 SF/Day

A = 35,000 Square Feet

ρ = 62.4 #/CF (water density)

C_p = 1 BTU/#/$^{\circ}$F (water heat capacity)

$$\Delta T_{max} = \frac{2 \times 10^{11}}{2 \times 62.4 \times 35{,}000\sqrt{0.12 \times 2.8 \times 10^8}} = 7.9^{\circ}F$$

Assume load centrally located, 6.5 miles from each bay

$$\Delta T_{Bay} = \Delta T_{max} e^{-\sqrt{0.06/10}\,|6.5|}$$

$$= 3.9^{\circ}F$$

*For a detailed discussion, see Lawler, J.P., and Leporati, J.L., "Receiving Water Temperature Distributions from Power Plant Thermal Discharges." Proceedings, 17th Southern Water Resources and Pollution Control Conference, Chapel Hill, North Carolina, (April 17, 1968)

100,000 #/day of this would be available for present conditions, and 32,000 #/day for stabilizing the BOD from future additional loads. Thus, the future additional load, at the 80% treatment level, could reach 160,000 #/day.

Other calculations of the nitrogeneous effect, in which the 5 day BOD/N ratio for municipal and industrial waste, the percentage nitrogen removal on treatment, and the stochiometric O_2/N ratio are considered, yielded estimates of the nitrogeneous demand on the Kill of 50,000 to 68,000 #/day. Furthermore, the parameters selected for decay rate and ultimate BOD may reflect a total effect rather than just a carbonaceous demand so that the net nitrogeneous effect may be less than described above.

Estimation of the potential nitrogeneous demand after treatment cannot be considered definitive without additional investigation. This would include determination of the type of decay kinetics to be expected, and estimation of the rate constants controlling these kinetics. Laboratory investigations, in which present and future Kill conditions were simulated, and in which measurements of oxygen uptake, nitrogen transformation, and CO_2 respiration were made, as well as evaluation of field measurements on Arthur Kill nitrogen, would be necessary.

VII. Waste Assimilation Capacity of the Hudson River

Introduction

Total loads and pollutant transport in each reach for the overall harbor complex model are shown in Figure 29. This model showed that less than 10% of the contaminants introduced into Upper New York Bay left the Bay through the Kill Van Kull, and that contaminant transfer between the Harlem and Hudson Rivers was negligible. Thus, a simpler Hudson River pollutant transport model was constructed.

This model is outlined on Figure 29 and includes only the lower Hudson River, the East River and Upper New York Bay. New Jersey wastes are discharged to the Bay and the Hudson River. The East River is included because it contributes a large waste load to the Bay and exercises a strong influence on contaminant concentration in the Bay. Mathematical development of this Hudson River model is similar to the development of the Arthur Kill model. Defining differential equations, integrated forms and boundary conditions are constructed as before. Details of the development are given in an earlier study report (1). The model is shown in Figure 38.

Model Verification

Machine computed BOD and DO distributions, using present loads, are compared to measured values of BOD and DO to verify model accuracy.

Quarterly average NYCDPW Harbor Survey data, June to September 1964, are used. Computed Hudson and East River BOD, DOD and DO concentrations are shown in Table 15. BOD comparisons for both Rivers are shown in Figure 39, Hudson River comparisons in Figure 1 and East River DO comparisons in Figure 40.

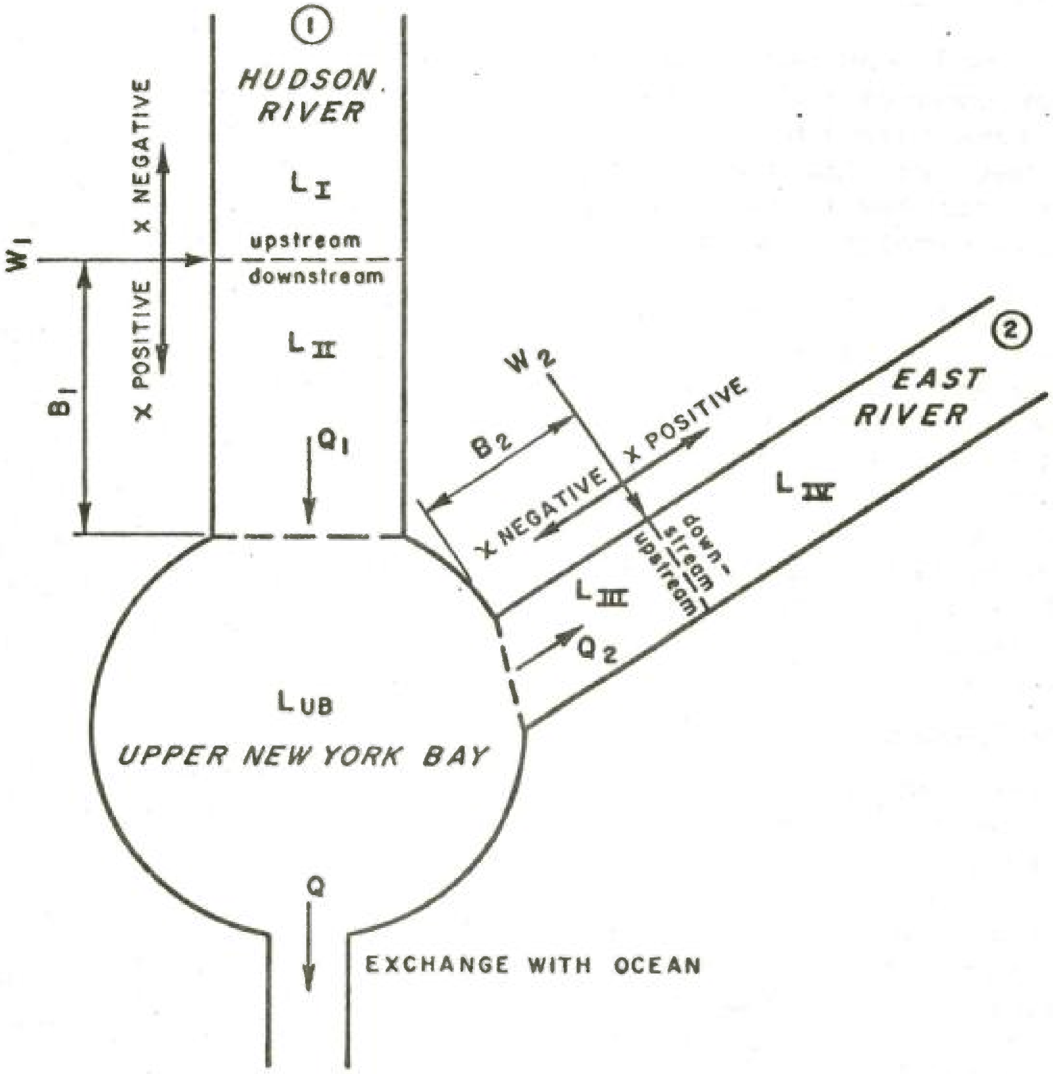
Figure 39 shows computed and measured values agree reasonably well through mile point 10 of both Rivers. Divergence in the values above mile point 10 may be due to the following factors:

1. The boundary condition for each River's upstream reach assumed concentration approached zero as distance increased. This is not true because BOD does exist in these upper reaches; the simplification eliminates an increasing exponential function, causing computed values to be low.

2. Computed concentrations represent average behavior over the cross-section and tidal cycle. Samples are taken at many points over the tidal cycle but generally

LOCAL MODEL FOR BOD & DISSOLVED OXYGEN TRANSPORT

HUDSON & EAST RIVERS & UPPER NEW YORK BAY



	HUDSON RIVER	EAST RIVER	UPPER NEW YORK BAY
DISPERSION COEFFICIENT	E_1	E_2	—
CROSS-SECTIONAL AREA	A_1	A_2	—
VELOCITY	U_1	U_2	—
UNIT DEOXYGENATION	K_1	K_2	K_3
UNIT REAERATION	K_21	K_22	K_23
EXCHANGE COEFFICIENT	—	—	R
VOLUME	—	—	V_3
WASTE LOAD	W_1	W_2	
DISTANCE TO BATTERY	B_1	B_2	

TABLE 15

BOD AND DO DISTRIBUTIONS IN THE HUDSON AND EAST RIVERS AS A RESULT OF THE PRESENT BOD DISCHARGE

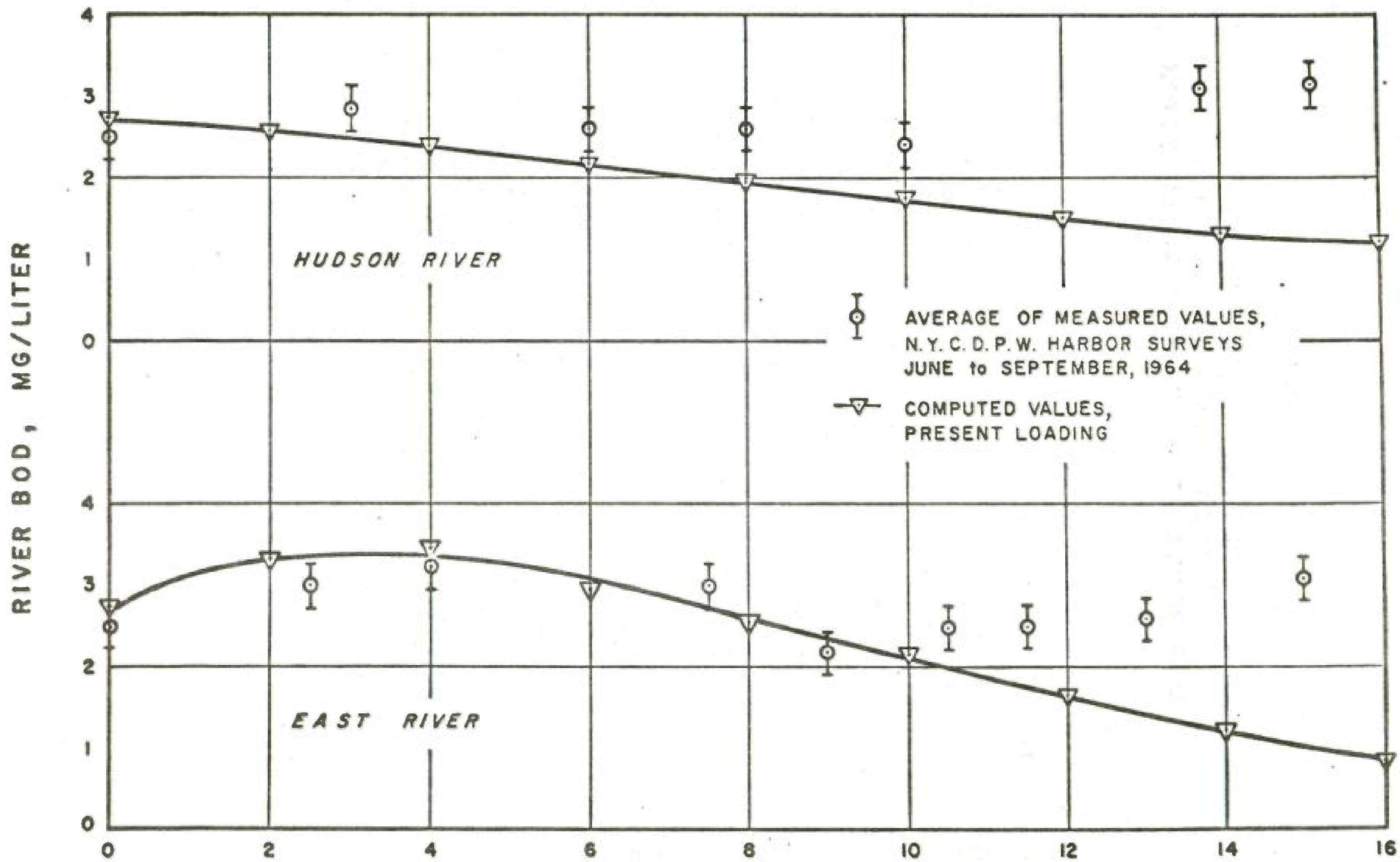
HEDO 16:59 NY TUE 10/25/66

HR DIS MILES	HR BOD PPM	HR DEF PPM	HR 02 PPM	HR 02 PERCENT SAT
0	2.72949	4.78495	3.21505	40.1881
5	2.29579	4.61998	3.38002	42.2502
10	1.71483	4.1859	3.8141	47.6763
15	1.24939	3.63079	4.36921	54.6151
20	.871155	3.05442	4.94558	61.8198
25	.600946	2.5208	5.4792	68.49
30	.451696	2.0624	5.9376	74.22
35	.391726	1.67111	6.32889	79.0112
40	.366364	1.35055	6.64945	83.1182
45	.222211	1.0584	6.9416	86.77
50	.134778	.812183	7.18782	89.8477
ER DIS MILES	**ER BOD PPM**	**ER DEF PPM**	**ER 02 PPM**	**ER 02 PERCENT SAT**
0	2.72949	4.78495	3.21505	40.1881
2	3.39995	5.50926	2.49074	31.1343
4	3.46106	5.90561	2.0944	26.1799
6	2.95848	5.97094	2.02906	25.3633
8	2.53997	5.78565	2.21435	27.6794
10	2.18572	5.41943	2.58057	32.2572
12	1.67876	4.92095	3.07905	38.4881
14	1.21939	4.35461	3.64539	45.5674
16	.856244	3.77793	4.22207	52.7758
18	.601245	3.22934	4.77066	59.6332
20	.422187	2.72913	5.27087	65.8859

TIME: 20 SECS.

HUDSON AND EAST RIVER ASSIMILATION CAPACITY ANALYSIS

COMPARISON OF MEASURED AND COMPUTED BOD DISTRIBUTIONS



DISTANCE FROM BATTERY, MILES

FIGURE 33

EAST RIVER ASSIMILATION CAPACITY ANALYSIS

COMPARISON OF MEASURED AND COMPUTED DISSOLVED OXYGEN DISTRIBUTIONS

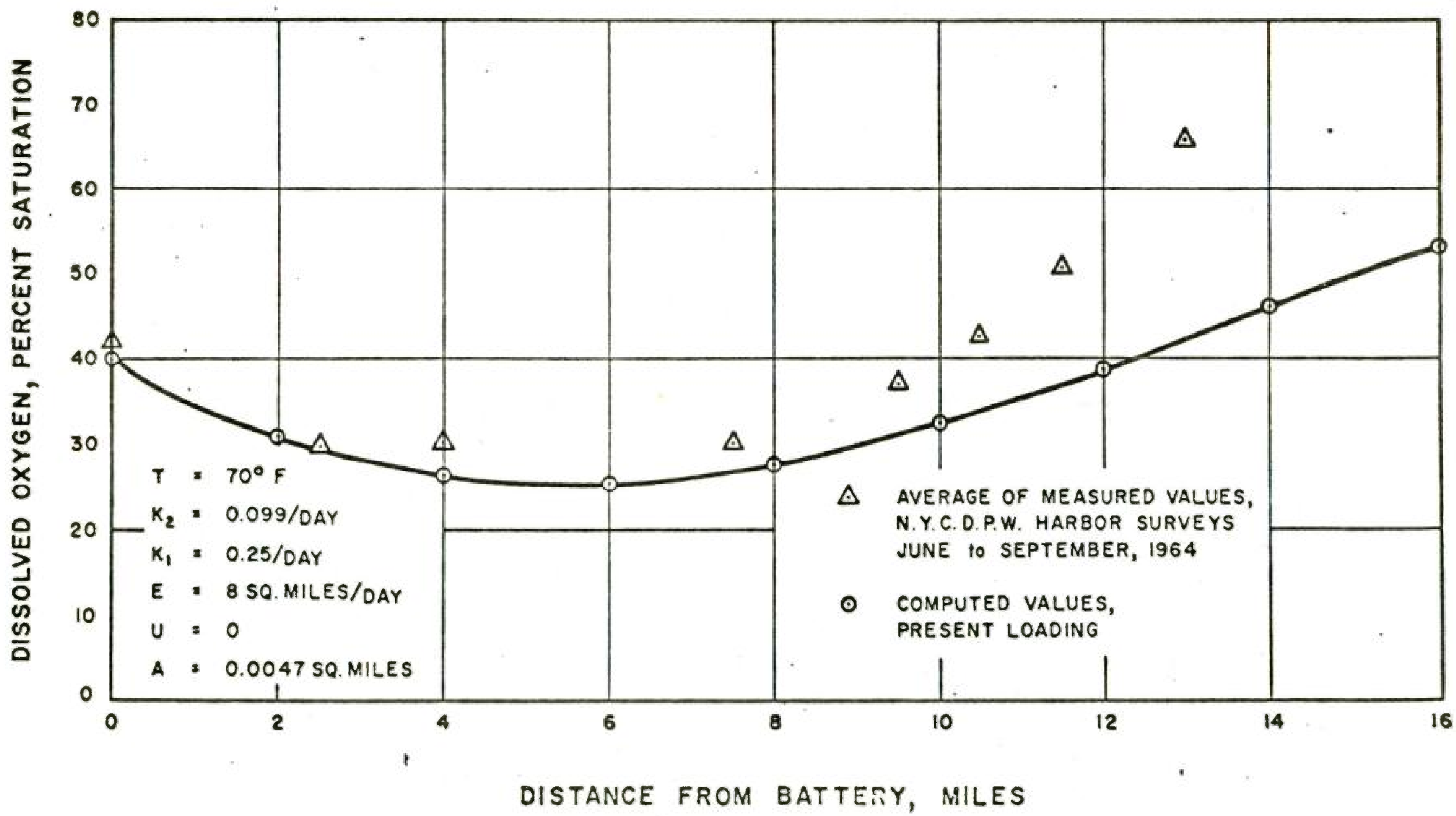


FIGURE 40

at the same point in space. Reportedly, some River sampling in the Hudson occurs near sewage outfalls; the BOD of such a sample will be higher than the area average.

Figure 39 shows that, near the River mouths, the East River BOD decreases toward the Bay, whereas the Hudson River BOD decreases in the upriver direction. Since East River pollutant transport depends on dispersion only, waste matter is carried from the East River into Upper Bay; i.e., in the direction of the decreasing concentration gradient. In the Hudson, under present loading conditions, convective pollutant transport is downstream; net transport will depend on the relative magnitudes of E, U, L and dL/dx.

Figure 1 shows good agreement between the measured and computed Hudson River dissolved oxygen profiles. Figure 40 indicates good agreement in the East River through mile point 10. Divergence in DO above this point may be due to model simplification. In order to avoid increasing mathematical complexity, upper East River area, which actually expands quite rapidly above mile point 10, was held constant. Once the actual area exceeds the constant value selected for model use, computed dissolved oxygen will be less than actual DO.

These results indicate the mathematical model will accurately predict long term River dissolved oxygen concentrations in the subject area. Divergence from measured oxygen distribution in the upper East River does not prevent prediction of lower Hudson River behavior. The upper East River is far from the subject area and does not control its behavior.

Effect of Individual Waste Discharges

Figure 41 shows the long term average Hudson River DOD distributions due to the Passaic Valley load alone, to the New York City load alone, and to the overall loading pattern delineated in Tables 3, 4 and 5.

The total load discharged to Upper New York Bay from the Passaic Valley Sewerage Authority was determined to be 650,000 #BOD/day. This causes a depletion of 25 percentage saturation units at the Battery, the critical minimum point, or 42% of the total depletion that occurs at this point. Average depletion over the lower 16 mile section of the River, due to PVSA effluent, is 20 percentage saturation units.

HUDSON RIVER

DISSOLVED OXYGEN DEFICIT DISTRIBUTION

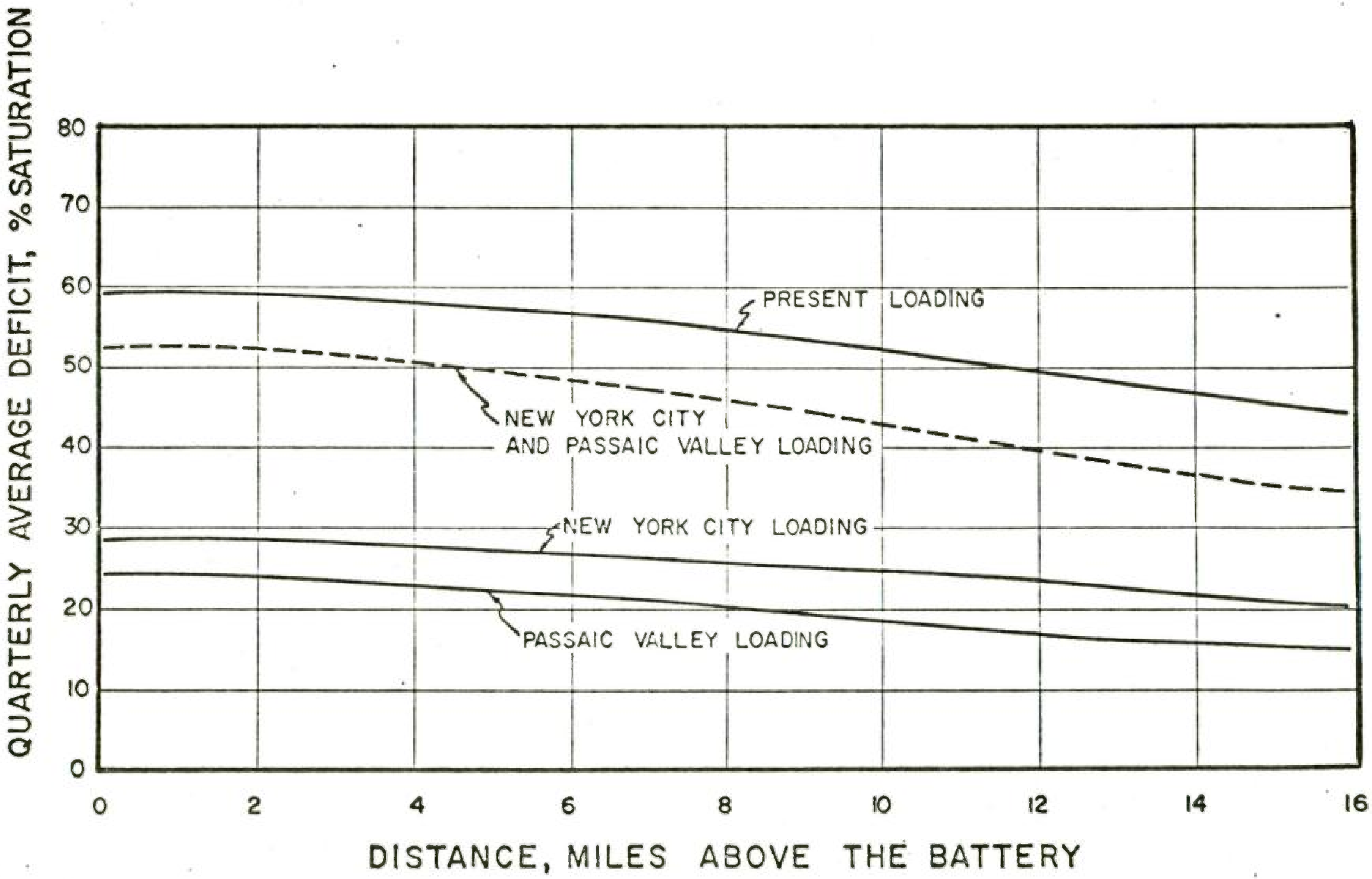


FIGURE 41

The total waste discharged from New York City to the Hudson River and Upper New York Bay is 363,000 #BOD/day. The New York City load causes a depletion of 28% saturation units, or 46.5% of the total depletion, at the critical point and an average depletion of 25% of saturation throughout the reach.

These results show that, of the present maximum oxygen deficit of 60% saturation in the New York Harbor area, 88.5% is accounted for by the New York City and Passaic Valley Sewerage Authority loads. The remainder is due chiefly to New Jersey loads in the near vicinity of New York Harbor. A negligible fraction is due to upper River loads.

Assimilation Capacity and Treatment Requirements

Computation of treatment estimates to meet stream minimums follows the procedure outlined for the Arthur Kill. NJSDH water quality standards require 50% DO saturation be maintained in the Hudson River and Upper New York Bay at all times. NYSDH standards require an average 50% DO saturation be maintained during any week and 3 ppm be maintained at any time.

As in the Arthur Kill analysis, the computed values represent the summer quarterly averages. Variation of individual Hudson River samples below the summer quarterly average during the 1964 Harbor Survey is shown in Table 16. A maximum variation of 24% saturation units occurred at 125th Street. Other variations were less than 20% of saturation. The value of 25% of saturation was selected as the expected difference after treatment between the minimum summer quarterly average and the minimum allowable DO. This value has been employed in this study to compute treatment estimates to meet regulatory agency standards for the Hudson River.

Figure 1 shows that under present loading conditions the maximum long term deficit occurs at the Battery and is 60% of saturation. The waste assimilation capacity of the receiving water body is defined as the quantity of organic matter which can be biologically stabilized without depressing dissolved oxygen below a prescribed minimum concentration. For a waterway characterized by a single source of waste, computation of assimilation capacity from results similar to those shown in Figure 1 is direct; the assimilation capacity is equal to the present loading times the ratio of the desired deficit to the actual deficit corresponding to the present loading.

In the case of a multi-source, multi-waterway system, the concept of assimilation capacity becomes more difficult to manage

TABLE 16

COMPARISON OF 1964 THREE MONTH
SUMMER AVERAGE DO WITH THE MINIMUM DO
OBSERVED AT ANYTIME IN THE HUDSON RIVER

	OBSERVED DO		
Station	Average % Saturation	Minimum % Saturation	Variation in % Saturation
Pier A	42	26	16
W. 42nd Street	37	25	12
W. 72nd Street	44	30	14
W. 125th Street	43	19	24
W. 155th Street	46	27	19
Spuyten Duyvil	52	34	18
Mt. St. Vincent	55	38	17

for several reasons. Waste loads are spread throughout the system and elimination of some may not alter the maximum deficit; for example, had waste loads from Rockland and Westchester Counties not been considered in the HEDO model, little difference would have been observed in the deficit at the Battery.

Treatment will not only lower the magnitude of the loads but will also alter the spatial loading configuration. Distributed untreated loads along the Hudson and East Rivers will be replaced by single sources of waste at the North River and Newtown Creek Treatment Plants. Such changes can cause a shift in the deficit pattern and even a relocation of the maximum deficit.

To utilize capacity in an optimum manner, a decided imbalance may result in the degree of waste treatment required at various outfalls. Unless costs are allocated among users on the basis of use, it is unlikely that treatment imbalance can be made acceptable, particularly to the group requiring the maximum degree of treatment.

For these reasons, the selection of a numerical value for the assimilation capacity of the lower Hudson system is not straightforward. Computation of deficit profiles for the levels of treatment proposed for this area, however, shows that the location of the maximum deficit does not change and that deficit is approximately proportional to change in total load.

The total present loading on the Hudson River-Upper Bay-East River system is 1,800,000 #/day. Of this, 1,200,000 #/day is discharged to the Hudson River and Upper Bay, and 600,000 #/day to the East River. Distribution between the Bay and the Hudson River, and between New York City, upstream New York State, and New Jersey sources is shown in Tables 3, 4 and 5.

An assimilation capacity has been computed for the entire system and for Hudson River-Upper Bay segment. Values have been obtained for both the NYSDH and the NJSDH standards. These results are shown in Table 17.

The values in Table 17 reflect the behavior of the constant parameter model for one set of parameter of flow, temperature, dispersion and decay. In addition, no consideration of the effect of nitrification and the relation of average to point behavior after treatment has been included. Studies currently in force will include some evaluation of these items. For these reasons, the values in Table 17 must be viewed as preliminary estimates of River behavior.

TABLE 17

SELECTION OF ASSIMILATION CAPACITY VALUES
FOR THE
LOWER HUDSON - UPPER BAY - EAST RIVER COMPLEX

	WATERWAY SYSTEM		
	Hudson River-Upper Bay		
	From NY-NJ Line	From Peekskill	Hudson River, Upper Bay and East River
Load, #/day BOD	1,100,000	1,200,000	1,800,000
Long Term Maximum Deficits			
- Present	60	60	60
- NYSDH	35	35	35
- NJSDH	25	25	25
System Capacity			
- NYSDH	650,000	700,000	1,000,000
- NJSDH	450,000	500,000	750,000

Maximum permissible long term deficit under NJSDH regulations was computed by subtracting the 25% saturation variation observed to exist between long term and "at any time" DO values (Table 16) from the "at any time" NJSDH DO standard of 50% saturation; i.e., to maintain 50% saturation at any time, long term average DO concentration must exceed 75%.

Maximum permissible long term deficit under NYSDH regulations was computed by converting the 3 ppm time requirement to an equivalent 40% DO saturation. To provide sufficient protection to meet this standard, the long term average minimum DO must be at least 65% saturation, and the corresponding maximum deficit no greater than 35%.

The original system included loads in the Hudson between Peekskill and the Narrows. Since these extreme upstream loads contribute little to the maximum deficit observed at the Battery, one set of computations has been made by eliminating this source from the system load. Of the three sets presented, the capacities associated with this computation most nearly represent the capacity available for assimilation of effluents discharged to the lower Hudson and Upper New York Bay.

Table 18 represents the distribution of available capacity between the two users of the River from the State Line to the Narrows.

Table 3 shows present New Jersey loading is approximately 730,000 #/day. Virtually all the New Jersey municipal waste treatment plants included in this estimate handle significant industrial wastes and are primary plants. An average BOD removal of 17%, found by measurement to apply to the PVSC plant (90% of the total New Jersey load), has been applied to this load to compute influent conditions. The value of 175,000 #/day for the New Jersey loads represents 80% treatment of the influent loading.

The New York loading of 175,000 #/day includes 20,000 #/day from the City of Yonkers and effluent loads from each of the New York City plants which are handling or will handle wastes currently discharged to either the Hudson River or Upper Bay. These effluent loads have been based on the expected degrees of treatment from each of the New York City plants. Detailed analyses of these are given in a previous report (1).

These results show that some 70 to 80% of the waste assimilation capacity may be utilized after 80% BOD removal of present loads. Some 20 to 30% reserve capacity remains for future loading. Should

TABLE 18

DISTRIBUTION OF CAPACITY BETWEEN
NEW YORK STATE AND NEW JERSEY USERS

	Total	New York	New Jersey
Loading, #/day after 80% treatment, each state	350,000	175,000	175,000
Under NJSDH Standards			
Available capacity, #/day	450,000	--	--
% of capacity used by each state after 80% treatment	--	40	40
% of capacity provided for future	20	10	10
Under NYSDH Standards			
Available capacity, #/day	500,000	--	--
% of capacity used by each state after 80% treatment	--	35	35
% of capacity provided for future	30	15	15

all the assimilation capacity be utilized for removal of present loads, on a 50-50 basis between States, required New Jersey removal would be 75%. Under these conditions, required removal for New York State is less readily calculated because of the different performance of each plant. However, on an overall basis, average treatment would be approximately 70%.

QUIRK, LAWLER & MATUSKY ENGINEERS

Nomenclature

A	Area of a section normal to the longitudinal axis of the estuary, L^2
D	Dissolved oxygen deficit, ML^{-3}
E	Longitudinal dispersion coefficient, L^2T^{-1}
K	First order reaction rate constant, T^{-1}
K_2	Rate of surface reaeration, T^{-1}
L	Concentration of non-conservative contaminant, ML^{-3}
L_w	Concentration of contaminant in waste flow, ML^{-3}
Q	Fresh water flow, L^3T^{-1}
Q_w	Waste flow, L^3T^{-1}
U	Average fresh water velocity, LT^{-1}
V	Volume, L^3
W	Single point source of waste, MT^{-1}
c	Concentration, ML^{-3} (used variously, for salt, contaminant, and DO)
c_s	Saturation concentration of dissolved oxygen in water, ML^{-3}
$\vec{j}$	Mass flux relative to mass average velocity, $ML^{-2}T^{-1}$
ℓ	Concentration of non-conservative contaminant, ML^{-3}
η	Mass flux relative to stationary coordinate axis, $ML^{-2}T^{-1}$
t	Time variable, T
$\langle V_x \rangle$	Area averaged estuarine velocity, LT^{-1}
x	Distance along the longitudinal axis of the estuary, L
y	Lateral coordinate, L
z	Vertical coordinate, L
$-$	Average value over volume

Nomenclature Continued

$\langle\;\rangle$ Average value of the variable over area A

$\overline{\langle\;\rangle}$ Average value of the area averaged variable over a tidal cycle

$\langle\;\rangle'$ Fluctuation of the instantaneous area averaged variable about the tidal smoothed, area averaged value

Γ Dimensionless concentration variable

ξ Dimensionless distance variable

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